Math 221/821 **Final Exam** **Dec. 14, 2000**

Name:______________________________ **Score:**____________

Instructions: You must show supporting work to receive full and partial credits.

1(16 pts) Find the general solution to the separable equation $\cos y \dfrac{dy}{dt} = t\sin y + 2t$. (Either an implicit or explicit solution is acceptable.)

2(16 pts) Solve the initial value problem to the 1st order linear equation $y' - \dfrac{2y}{t} = 2t^2, y(1) = 2$. (You must solve it by the method for 1st order linear equations. Any other way will receive significantly small credit.)

3(18 pts) (a) Use the Euler method to approximate the solution to the initial value problem $y' = t - t^2y^2, y(1) = 1$ over the interval $1 \le t \le 2$ at these points: $t = 1, 1.25, 1.5, 1.75, 2$.
(b) Sketch your approximating solution.

4(16 pts) The characteristic equation for a homogeneous, 3rd order linear, and constant coefficient equation $ay''' + by'' + cy' + dy = 0$ is factored as $ar^3 + br^2 + cr + d = (r-1)(r^2 + r + 1) = 0$. Find the general solution to the differential equation.

5(18 pts) Use the method of undetermined coefficients to find a suitable **form** for a particular solution $Y(t)$ to the equation $y'' + 2y' + y = 1 - 2t^2e^t\cos 2t + 2e^{-t}$. (Do not solve for the coefficients!)

6(18 pts) Use the method of variation of parameters to find a particular solution to the equation $y'' + by' + cy = e^{2t}$ given that $y_1(t) = 1, y_2(t) = e^t$ are two solutions to the homogeneous equation $y'' + by' + cy = 0$. (Solution by any other method will receive no points.)

7(18 pts) Consider a system of cooperating species $x' = x(2 - x + y),\ y' = y(1 - y + ax)$.
(a) Sketch a phase portrait of the system when $a = 0.5$.
(b) What is the parameter a value so that the coexisting equilibrium solution (x_0, y_0) will has the value $x_0 = 6$?

8(18 pts) (a) Verify that $e^{xy} + y = x + 1$ is an implicit solution to the differential equation

$$\frac{dy}{dx} = \frac{e^{xy} - y}{e^{xy} + x}.$$

(b) The direction field of a differential equation is given by the graph. Sketch an approximating solution with the initial point $y(0) = 2$.
(c) What is your estimate for the value of $y(2)$?

8(18 pts) Find the following Inverse Laplace Transforms:
(a) $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2 + s}\right\}(t)$.
(b) $\mathcal{L}^{-1}\left\{\dfrac{2s}{s^2 - 6s + 11}\right\}(t)$.

10(10 pts) Use the definition for Laplace Transforms to find $\mathcal{L}\{2\}(s)$. (Any other method will receive no point!)

11(16 pts) (a) Express the function $f(t) = \begin{cases} 0, & 0 \le t < 2 \\ 2 - t, & 2 \le t < 3 \\ 2, & 3 \le t \end{cases}$ in terms of a combination of two unit step functions.
(b) Find the Laplace Transform, $\mathcal{L}\{f(t)\}$, of the function $f(t)$ of (a).

12(18 pts) Find the Laplace Transform $\mathcal{L}\{y(t)\}(s)$ of the solution $y(t)$ to the initial value problem

$$y'' + 5y' + 6y = e^{-3t}, \quad y(0) = 3, y'(0) = -3.$$

(Do not solve for $y(t)$.)

END