Final Exam Outline

1. Vectors in $\mathbf{V}_2$ and $\mathbf{V}_3$.*

a. Basic properties. Magnitude, vector algebra, parallel vectors, etc.

b. The dot product. The angle between vectors. Orthogonal vectors.

c. The cross product. The area of a parallelogram. The volume of a parallelpiped.

2. Vector-valued functions.

a. Functions from $\mathbf{R}$ to $\mathbf{V}_2$ or $\mathbf{V}_3$.

b. Oriented curves traced by vector-valued functions, in particular, circles, ellipses, lines and line segments.

c. The tangent (derivative) vector to a curve traced by a vector-valued function.

d. The arclength integral.

3. Motion in space.

a. Position, velocity, speed and acceleration.

b. Newton's second law.

c. Projectile motion.

4. Functions of several variables.

a. Functions taking $\mathbf{R}^2$ or $\mathbf{R}^3$ to $\mathbf{R}$. Their domains and ranges.

b. Contours and traces of functions of two variables.

c. Level surfaces of functions of three variables.

5. Partial derivatives.

a. Definition and calculation of partial derivatives of functions of two and three variables.

b. Higher-order partial derivatives.

c. Equality of certain partial derivatives, e.g. f_{xy} and f_{yx}.

6. Linear approximation.

a. The tangent plane to the graph of a function of two variables.

b. Differentials. The principle of (local) linear approximation, versions I and II.

c. Linear approximation and differentials of functions of three variables.

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* You won't be tested *explicitly* on this material.

7. The chain rule.

- **a**. The chain rule for various compositions: $z(t) = f(x(t), y(t))$, $u(x,t) = f(\theta(x,t))$, $w(p,q) = f(x(p,q), y(p,q))$, etc. Tree diagrams.
- **b**. Second derivative with the chain rule.
- **c**. Implicitly defined functions. Partial derivatives of implicit functions.

8. Gradients and directional derivatives.

- **a**. The gradient of a function of several variables.
- **b**. The derivative of a function at a point P in a direction $\vec{v}$.
- **c**. Properties of the gradient. Direction and rate of most rapid increase. The orthogonality of ∇g to the contour surfaces (or curves) of g.

9. Extrema of functions of several variables.

- **a**. Relative (or local) extrema and saddle points.
- **b**. Critical points.
- **c**. The second derivative test for functions of two variables.

10. Constrained optimization.

- **a**. The Lagrange multiplier method for identifying constrained extrema.
- **b**. Finding absolute extrema over a region bounded by a curve or surface.

11. Double integrals.

- **a**. The double integral over a rectangular and nonrectangular regions.
- **b**. Iterated integrals over rectangles and regions bounded by curves. Fubini's theorem. Changing the order of integration.
- **c**. Applications of the double integral. Volume, area, laminar mass, surface area.
- **d**. Double integrals in polar coordinates.

12. Triple integrals.

- **a**. Triple integrals over boxes.
- **b**. Iterated integrals with $dV = dx\,dA$, $dV = dy\,dA$ and $dV = dz\,dA$. Changing the order of integration.
- **c**. Applications of the triple integral: Volume, mass.
- **d**. Triple integrals with cylindrical coordinates.
- **e**. Triple integrals with spherical coordinates.

13. Vector fields.

- **a**. Two and three dimensional vector fields.
- **b**. Conservative, or gradient fields. Potential functions.
- **c**. The curl of a vector field. The curl test for conservative fields.
- **d**. Finding the the potential of a conservative field.

14. Line integrals.

- **a**. Line integrals of functions.
- **b**. Line integrals of vector fields over oriented curves. Calculation of the work done traversing a curve in a force field.
- **c**. Evaluation of line integrals by parametrization.
- **d**. The fundamental theorem of line integration.

15. Surface integrals.

- **a**. Integrals of vector fields over surfaces (flux integrals).
- **b**. Calculation of flux integrals.
- **c**. The divergence of a vector field.
- **d**. The divergence theorem.

16. Stokes' theorem and Green's theorem.

- **a**. Stokes' theorem.
- **b**. Green's theorem as the two-dimensional version of Stokes' theorem.