Math 208 **Test 3** **April 25, 2003**

Name:_____ Score:_____

Instructions: You must show supporting works to receive full and partial credits.

1(15 pts) (a) Find the work done by the force field $\vec{F} = 5\vec{i} - 9\vec{j} + 2\vec{k}$ from $(1, 0, 3)$ to $(1, 1, 4)$.

(b) Find the flux of the vector field $\vec{v} = 3\vec{i} + \vec{j} - 5\vec{k}$ through the rectangle with vertices $(1, 1, 0), (0, 1, 0), (0, 0, 1), (1, 0, 1)$, oriented upward.

2(20 pts) Let $\vec{F}(x, y, z) = (x^2 - y)\vec{i} + (y^2 - z)\vec{j} + (z^2 - x)\vec{k}$

(a) Find the divergence of $\vec{F}(x, y, z)$ at the point $(1, 1, 1)$.

(b) Find the curl of $\vec{F}(x, y, z)$ at the same point $(1, 1, 1)$.

(c) Find the circulation density $\text{circ}_{\vec{n}}\vec{F}(1, 1, 1)$ of $\vec{F}$ and in the direction from $(1, 1, 1)$ toward $(1, 0, 1)$.

3(20 pts) (a) Find the line integral of the vector field $\vec{F}(x, y) = x\vec{i} - y\vec{j}$ along the unit circle $x^2 + y^2 = 1$ from the point $(1, 0)$ to $(0, 1)$.

(b) **Set up** an iterated double integral for the flux of the vector field $\vec{F}(x, y, z) = z\vec{i} + x\vec{j} + y\vec{k}$ through the part of the parabola $S : z = 1 - x^2 - y^2$ lying above the xy-plane, oriented upward. Simplify but **DO NOT** evaluate the integral.

4(25 pts) (a) Use a derivative test to show that the vector field $\vec{F}(x, y, z) = (yz + z)\vec{i} + (xz + 1)\vec{j} + (xy + x)\vec{k}$ is conservative.

(b) Find a potential function f for $\vec{F}$.

(c) Find the line integral $\int_{(1,2,0)}^{(0,1,3)} \vec{F} \cdot d\vec{r}$ using the Fundamental Theorem of Line Integral.

5(10 pts) Use Stoke's Theorem to find the line integral integral of $\vec{F}(x, y, z) = y\vec{i} + z\vec{j} + x\vec{k}$ around the square with vertices $(0, 0, 2), (1, 0, 2), (1, 1, 2), (0, 1, 2)$, on the $z = 2$ plane, oriented counterclockwise when viewed from above.

6(10 pts) Use the Divergence Theorem to find the flux of the vector field $\vec{F}(x, y, z) = (x - y)\vec{i} + (y - z)\vec{j} + (z - x)\vec{k}$ through the sphere of radius 5, centered at the origin, oriented outward.

Bonus(3 pts) The State Bird of Nebraska is: _____

END