Math 208 Fall 2013 **Exam 3**

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) (a) Find a parameterized equation for the line from $P(1,2,3)$ to $Q(3,2,1)$.

(b) Find the flux of the vector field $\vec{F} = \vec{i} + 2\vec{j} + 3\vec{k}$ through the rectangle with vertices $(0,0,1), (1,0,0), (1,2,0), (0,2,1)$, oriented upward.

2(15pts) Let C be the helix $x(t) = 2\cos(t), y(t) = 2\sin(t), z(t) = t$ for $0 \leq t \leq 2\pi$. Find the line integral of the vector field $\vec{F}(x,y,y) = -y\vec{i} + x\vec{j} + z\vec{k}$ along C.

3(20pts) Let $\vec{F}(x,y,z) = xy\vec{i} + yz\vec{j} + zx\vec{k}$

(a) Find the divergence of $\vec{F}(x,y,z)$ at the point $(1,2,3)$.

(b) Find the curl of $\vec{F}(x,y,z)$ at the same point $(1,2,3)$.

(c) Find the circulation density $\text{circ}_{\vec{n}}\vec{F}(1,2,3)$ of $\vec{F}$ and in the direction of $\vec{v} = \langle 1,1,1 \rangle$.

4(15pts) If the curl of a vector field $\vec{F}(x,y,z)$ is given as $\text{curl}\vec{F}(x,y,z) = (y-x)\vec{i} + x\vec{j} + z\vec{k}$. Use Stoke's Theorem to find the line integral of $\vec{F}(x,y,z)$ around the triangle with vertex $(0,0,2)$, $(2,0,0)$, $(0,2,0)$ on the plane $z = 2 - x - y$, oriented counterclockwise when viewed from above.

5(20pts) (a) Determine if the vector field $\vec{F} = \langle y, x + 2y \rangle$ is conservative by finding if it has a potential function f.

(b) Find the line integral $\int_C \vec{F} \cdot d\vec{r}$ of the vector field along the path C which is part of an ellipse from $(1, 0)$ to $(0, 2)$. (Use the Fundamental Theorem of Line Integral if possible.)

6(15pts) Use the Divergence Theorem to find the flux of the vector field $\vec{F}(x, y, z) = (x + y^2)\vec{i} + (y + z^2)\vec{j} + (z + x^2)\vec{k}$ through the surface of the solid cylinder: $x^2 + y^2 = 2, 0 \leq z \leq 1$, centered along the z-axis and oriented outward.

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2 Bonus Points: What is the first day of this year's winter?