Name: —————————————————   **Score:** ———————————————————
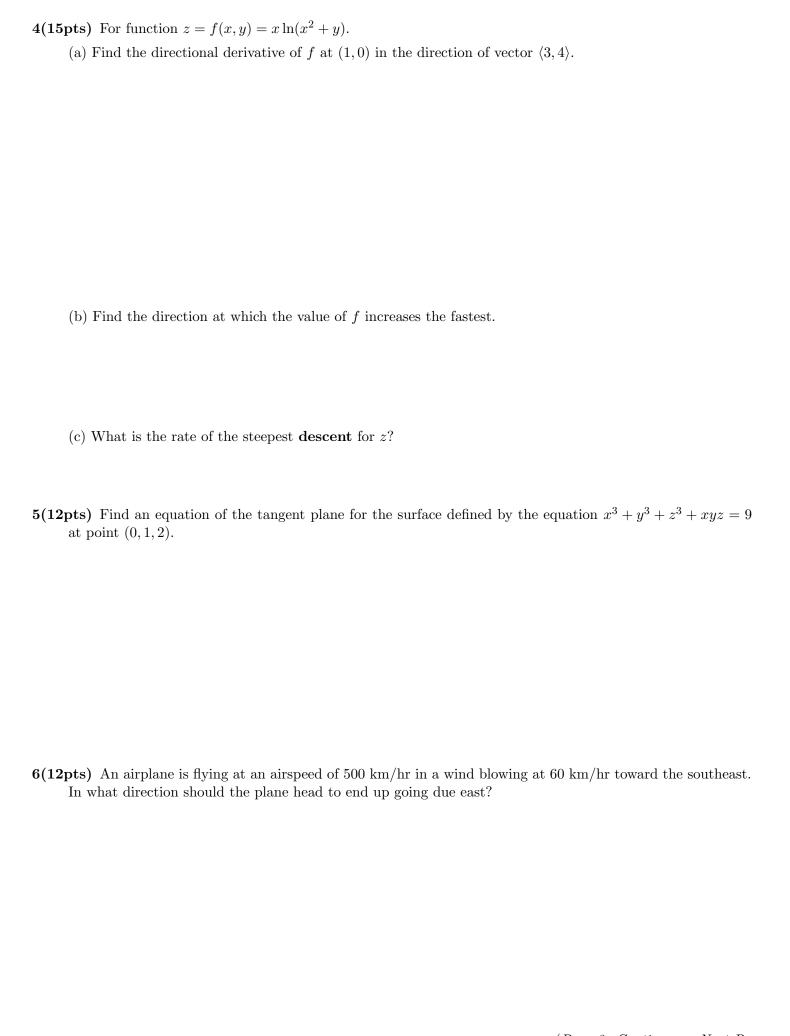
Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(12pts) Given two points $P = (1, 0, 2)$, $Q = (0, 1, 2)$, find the following.

(a) The distance between them.

(b) The unit vector from P to Q.

2(12pts) For two vectors $\vec{u} = \langle 1, 2, 0 \rangle$, $\vec{v} = \langle 1, 1, 1 \rangle$, find the projection of $\vec{u}$, $\vec{u}_{\text{parallel}}$, in the direction of $\vec{v}$.

3(10pts) For $\lim\limits_{(x,y) \to (1,0)} \dfrac{y}{x + y - 1}$, find the limit if exists. If the limit does not exist, explain why not.

4(15pts) For function $z = f(x, y) = x \ln(x^2 + y)$.

(a) Find the directional derivative of f at $(1, 0)$ in the direction of vector $\langle 3, 4 \rangle$.

(b) Find the direction at which the value of f increases the fastest.

(c) What is the rate of the steepest **descent** for z?

5(12pts) Find an equation of the tangent plane for the surface defined by the equation $x^3 + y^3 + z^3 + xyz = 9$ at point $(0, 1, 2)$.

6(12pts) An airplane is flying at an airspeed of 500 km/hr in a wind blowing at 60 km/hr toward the southeast. In what direction should the plane head to end up going due east?

7(12pts) For a function $z = f(x, y)$, you are given its partial derivatives:

$$\frac{\partial f(x, y)}{\partial x} = \frac{y}{(x + 2y)^2}, \quad \frac{\partial f(x, y)}{\partial y} = \frac{-x}{(x + 2y)^2}.$$

Find $\dfrac{\partial z}{\partial u}$ if in addition $x = u^2 + v^2$, $y = uv$.

8(15pts) For function $z = f(x, y) = xe^{2xy}$.

(a) Find the linear Taylor polynomial of f about $(1, 0)$.

(b) Find the 2nd partial derivative functions f_{xx}, f_{xy}, f_{yy}.

(c) Find the quadratic Taylor polynomial of f about $(1, 0)$.

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2 Bonus Points: Who discovered Calculus? (a) Albert Einstein. (b) Issac Newton. (c) Benjamin Franklin. (d) Thomas Edison.