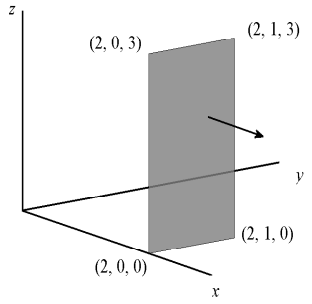
Math 208 Fall 2012 **Exam 6**

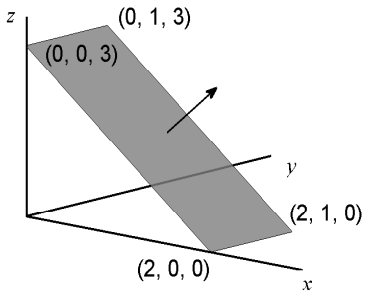
Name: _____ **Score:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(12pts) Find the flux of the vector field $\vec{F}(x, y, z) = 2\vec{i} + \vec{j} + 3\vec{k}$ through the surface of (a) and (b), respectively as shown.



(a)



(b)

2(16pts) Let S be part of the paraboloid $z = 9 - x^2 - y^2$ inside the rectangle cylinder $0 \le x \le 1,\ 0 \le y \le 2$. Find the flux of the vector field $\vec{F}(x, y, z) = \langle x, y, 2z \rangle$ through S which is oriented upward.

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3(12pts) Let $\vec{F} = \dfrac{-y}{x^2+y^2}\vec{i} + \dfrac{x}{x^2+y^2}\vec{j} + (x+y+z)\vec{k}$ and P be the point $(1,2,3)$.

(a) Find the divergence of $\vec{F}$ at point P.

(b) Find the curl of $\vec{F}$ at point P.

4(12pts) Suppose the curl of a vector $\vec{F}$ at point $Q(1,0,-1)$ is $\text{curl}\vec{F}(1,0,-1) = \langle 1,2,3\rangle$.

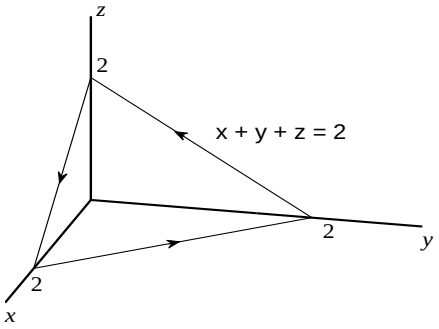
(a) Find the unit direction, $\vec{n}$, at which the circulation density $\text{circ}_{\vec{n}}\vec{F}$ of $\vec{F}$ at point Q is maximal and find the maximum circulation density.

(b) Find the circulation density $\text{circ}_{\vec{u}}\vec{F}$ of $\vec{F}$ at point Q and in the direction of $\vec{u} = \langle 1,1,1\rangle$.

(*Page 2. Continue on Next Page ...*)

5(14pts) Find the flux $\vec{F}$ through the closed cylinder of radius 2, centered around the z-axis, with $1 \leq z \leq 3$, if $\vec{F} = \langle x + 3y\ln(2yz+1),\ 2y + x^{\sin z},\ 2z + e^{x^2} \rangle$. (Use Divergence Theorem.)

6(14pts) Let $\vec{F}$ be a vector field whose curl is given as curl$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$. Let C be the boundary of the plane $x + y + z = 2$ in the first octant as shown. Find the line integral $\oint_C \vec{F}(\vec{r}) \cdot d\vec{r}$. (Use Stoke's Theorem.)



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2 Bonus Points: True or False: The Huskers has won the Big Ten football championship.