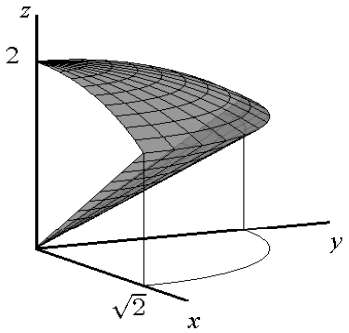
Math 208 Fall 2012 **Exam 4**

Name: _____ **Score:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(16pts) A solid region W is bound by the sphere $x^2+y^2+z^2=4$, inside the cone $z=\sqrt{x^2+y^2}$ in the first octant as shown. Without evaluating any of the iterated integrals below, set up the triple integral $\int_W z\sqrt{x^2+y^2+z^2}dV$ in

(a) the Cartesian coordinate in $dzdydx$.



(b) the cylindrical coordinate.

(c) the spherical coordinate.

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2(16pts) A double integral is set up in the polar coordinate $\int_{\pi/4}^{\pi/2} \int_{0}^{3/\sin\theta} r^2 dr d\theta$.

(a) Sketch the region of integration.

(b) Set up the integral in the Cartesian coordinate in the order of $dxdy$. (**Do not evaluate the integral.**)

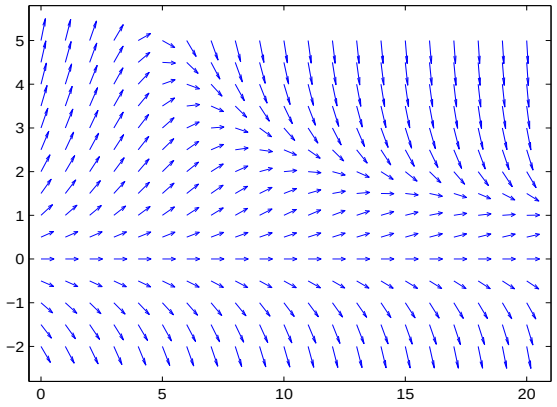
3(16pts) (a) The center of a disk moves on the z-axis at a speed of 2 cm per second. Write a parametric equation for the center of the disk when it is initially at the origin $(0, 0, 0)$.

(b) A spot on the rim of the disk is initially at the point $(1, 0, 0)$, and the disk rotate around the z-axis clockwise one revolution per second while its center moves on the z-axis. Write a parametric equation for the spot of the disk at any time $t \geq 0$ with respect to the reference origin.

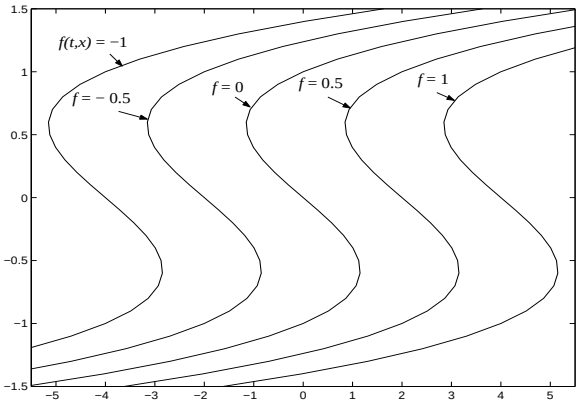
(c) Find the vector of acceleration for the spot.

(*Page 2. Continue on Next Page ...*)

4(16pts) (a) A vector field is given as shown. Sketch a flow line that starts at the point (0,1). (b) Some level curves of a function $f(t,x)$ are given as shown. Let $\vec{v}(t,x)$ be the gradient vector field of f. Choose two points on each of the level curves and then sketch $\vec{v}(t,x)$.



(a)



(b)

5(16pts) A vector field is given as $\vec{F}(x,y) = 2\vec{i} + x\vec{j}$.

(a) Find a system of differential equations for the flow lines of $\vec{F}(x,y)$

(b) Verify if the parameterized curve $\vec{r}(t) =< t,\ t^2 >$ is a flow line for the vector field.

(c) Verify if the parameterized curve $\vec{r}(t) =< 2t + C_1,\ t^2 + C_1 t + C_2 >$ is a flow line for any pair of constants C_1, C_2.

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2 Bonus Points: True or False: There are 12 schools in the Big XII Conference.