NAME:________________________

Instructor's name:____________________

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Value	10	12	12	20	10	15	10	12	14	10	18	10	10	12	11	14	200
Score																	

This exam should have pages; please check that it does. Show all work that you want considered for grading. Calculators are allowed, but **an answer will only be counted if it is supported by all the work necessary to get that answer.** Simplify as much as possible, except as noted: for example, don't write $\cos(\pi/4)$ when you can write $\sqrt{2}/2$. Also, give exact answers only, except as noted; for example, don't write 3.1415 for π. No cheating.

---

1. (10 points)

   (a) Sketch the region of integration of

$$\int_1^{e^8} \int_0^{\ln x} f(x,y)\,dy\,dx$$

   (b) Switch the order of integration. Do not evaluate.

2. (12 points) Let S be the surface given by

$$x^2 + z^2 + y = 12$$

and R be the point on the surface with coordinates $(1, 2, 3)$.

(a) Find a vector that orthogonal to the surface S at the point R.

(b) Write down an equation of the tangent plane to S at R.

3. (12 points) Use polar coordinates to evaluate the following integral:

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2 + y^2 + 1)\, dx\, dy.$$

4. (20 points) Let $f(x, y) = x^2y - 2xy + (1/2)y^2$.

   (a) Find all critical points of f.

   (b) Classify the critical points of f as local maximum, local minimum or saddle point.

5. (10 points) Use Green's Theorem to find $\int_C \vec{F}(x,y) \cdot d\vec{r}$, where $\vec{F}(x,y) = (\sin x^2 + 2y)\vec{i} + (y^2 + 5)\vec{j}$ and C is the circle $x^2 + y^2 = 2$ with the counterclockwise orientation.

6. (15 points) At what (x, y) does

$$x^{\frac{1}{10}} y^{\frac{9}{10}}$$

take on its maximum value subject to the constraint $x + 2y = 3$.

7. (10 points) Let

$$f(x, y) = x^2 + xy - y^2$$

and P be the point in the xy-plane with coordinates $(1, 2)$.

(a) What is the direction in which f increases most rapidly at P. What is the rate of increase in this direction?

(b) What is the rate of change of f in the direction from P to the point Q with coordinates $(-4, 14)$.

8. (12 points) Use spherical coordinates to compute the integral

$$\int_Q z\, dV,$$

where Q is the set of all (x, y, z) in the first octant which satisfy $x^2 + y^2 + z^2 \leq 1$.

9. (14 points) Let

$$f(x,y) = \frac{2xy}{x^2+y^2}.$$

(a) Draw the level curves $f(x,y) = 0$ and $f(x,y) = 1$.

(b) Does

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

exist? Explain your answer.

10. (10 points) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F}(x,y,z) = x\vec{i} + z\vec{j} + (2x+y)\vec{k}$, where C is the line segment from from $(-1,0,0)$ to $(2,1,-1)$.

11. (18 points) Let $\vec{F}(x,y) = (1 + 2x + xy^4)\vec{i} + (2x^2y^3 + 5y)\vec{j}$.

(a) Verify that the vector field $\vec{F}(x,y)$ is a conservative vector field.

(b) Find a potential function f for $\vec{F}(x,y)$.

(c) Find the $\int_C \vec{F}(x,y) \cdot d\vec{r}$, where C is that part of the parabola $x = 2y^2$ from $(2,1)$ to $(2,-1)$.

12. (10 points) Let

$$z = f(x,y), \qquad x = 4\sqrt{u}, \qquad y = u^2 + v + 1.$$

Calculate $\frac{\partial z}{\partial u}$ when $u = 4$ and $v = -10$ if it is known that

$$\left.\frac{\partial z}{\partial x}\right|_{(8,7)} = 3, \qquad \left.\frac{\partial z}{\partial y}\right|_{(8,7)} = 5, \qquad \left.\frac{\partial z}{\partial x}\right|_{(1,8)} = 7, \qquad \left.\frac{\partial z}{\partial y}\right|_{(1,8)} = 9.$$

13. (10 points) Let S be a disk of radius 2 in the plane $2x - 2y + 3z = 6$, oriented with upward-oriented normals. Compute the flux of $\vec{F}(x, y, z) = 3\vec{i} - \vec{j} + \vec{k}$ over S.

14. (12 points) Let S be the portion of the surface $z = x^2 - 2y$ with $-1 \leq x \leq 3$ and $0 \leq y \leq 2$, oriented with downward normals. Compute the flux of $\vec{F}(x, y, z) = x\vec{i} + 2z\vec{k}$ over S.

15. (11 points) Let S be the *closed* surface, oriented with outward normals, which is boundary of the cylindrical solid bounded by the cylinder $x^2 + y^2 = 9$ on the sides, $z = 0$ on the bottom, and $z = 4$ on the top. Compute $\int_S \vec{F} \cdot d\vec{A}$ when $\vec{F} = (x^2 z - 2x)\vec{i} + (-z^3 - 3y)\vec{j} + (2z - xz^2)\vec{k}$.

16. (14 points). Use Stokes' Theorem to find $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y, z) = 3y\vec{i} + (y^3 - 4z)\vec{j} + 2x\vec{k}$ and C is the square in the plane $y = 2$ with verticies $(0, 2, 0)$, $(0, 2, 1)$, $(1, 2, 1)$ and $(1, 2, 0)$, oriented counterclockwise when viewed from above.