Project: Change of Variables

Due Nov. 16

Before reading further you should read section 15.8, Focus on Theory, in our book.

Now that you have read 15.8, I will make a few comments on two typical situations where one makes a change of variable.

Situation 1. When the region is inconvenient in the original variables, sometimes a change of variable will lead to a much more convenient region. You have already seen how this works with polar coordinates. Here is another simple example: let **R** be the region below.

While this region can be described by two integrals in the x, y plane, we can transform it to an even easier region in the u, v plane by the change of variables $x = (u+v)/2, y = (v-u)/2$. Try integrating

$$\int_R xy \; dA$$

with this change of variable, but you don't need to hand it in.

Situation 2. Sometimes the integrand is really messy in the original variables, and a change of variables will simplify it quite a bit. For instance, suppose we wish to integrate

$$\int_R \sin(xy)\cos(x+y) dA.$$

Without a change of variable, this is very difficult. An obvious change of variable to try is $u = xy, v = x + y$. Of course, this change of variable might transform the region of integration into something uglier than the original region, but at least there would be a fighting chance of evaluating the integral.

Now compute the following integrals using a change of variable. You can of course check these integrals numerically if you'd like:

1. Evaluate

$$\int_{\mathbf{R}} x^2 y^2 dA,$$

where **R** is the region in the first quadrant between the curves $y = 1/x, y = 2/x, y = x, y = 3x$.

2. Let **R** be the ellipsoid

$$14x^2 + 4y^2 + 10z^2 - 8xy + 16xz = 36.$$

Use a change of variable to evaluate

$$\int_{\mathbf{R}} (x - z) dV.$$

Hint: See what happens with the substitution

$$u = x - y, v = x - z, w = x + z.$$

3. Evaluate

$$\int_{\mathbf{R}} e^{\frac{y-x}{y+x}} dA,$$

where **R** is the region inside the triangle with corners $(0,0), (1,1)$ and $(2,0)$.