Math 208 Summer II 2006 **Exam 2**

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(10pts) (a) Find the directional derivative of function $z = f(x, y) = x^2 y$ at the point $(1, 2)$ in the direction of the vector $\langle -3, 4 \rangle$.

(b) Find the direction at $(1, 2)$ at which the function decreases the most rapidly.

(c) Find the largest rate of change of the function at the point $(1, 2)$.

2(6pts) Consider the composition function $z = f(x(t), y(t))$. If $x(t) = t + t^2, y(t) = t^4$ and $f_x(2, 1) = -2, f_y(2, 1) = 3$, what is $\dfrac{dz}{dt}$ at $t = 1$?

3(6pts) Find the tangent plane of the ellipsoid $x^2 + y^2 + 2z^2 = 7$ at the point $(2, -1, 1)$.

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4(12pts) Find all critical points of $f(x, y) = x^4 + y^4 - 4xy$ and use the second derivative test to classify the points.

5(15pts) The temperature of a metal plate is given by $T(x, y) = \frac{300}{1+(x-1)^2+y^2}$, for points (x, y) on the circular plate defined by $x^2 + y^2 \leq 4$. Use Lagrange multiplier method to find the maximum and minimum temperatures on the edge of the plate.

6(12pts) Evaluate the integral $\int_0^1 \int_{\sqrt{y}}^1 \cos x^3 dxdy$ by changing the order of integration.

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7(15pts) (a) Sketch the solid over which the iterated triple integral $\int_0^2 \int_0^{\frac{6-3z}{2}} \int_0^{6-2y-3z} f(x,y,z)\,dxdydz$ is set.

(b) Change the order of the iterated integral to $dzdydx$.

8(10pts) Set up an iterated integral in polar coordinate for the area of a region outside the unit circle, $x^2 + y^2 = 1$, and insider another, $x^2 + y^2 = 2y$. (Needed special fact: $\sin\frac{\pi}{6} = \frac{1}{2}$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$. Do not evaluate the integral.)

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9(14pts) A solid Q is bounded these surfaces: $z = 1 - x^2, \ z = 0, \ y = 0, \ y + z = 2$. Find the total mass if $\rho(x, y, z) = x^2$ is the density function. (Suggestion: Set up the triple integral in the order of $dydzdx$.)

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2 Bonus Points: The gradient of a function is: (a) a number, (b) a vector, (c) a curve on the surface, (d) all above. (... *The End*)