Math 208 Summer II 2006 **Exam 1**

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(10pts) Three points in the space are given: $P(0,1,2), Q(1,2,3), R(-1,-1,1)$.

(a) Find the angle between the vectors $\vec{PQ}, \vec{PR}$.

(b) Find the projection of vector $\vec{PR}$ on $\vec{PQ}$: $\mathbf{proj}_{\vec{PQ}}\vec{PR}$.

2(6pts) Find the equation of the plane that goes through the point $(1,0,-2)$ and is parallel to another plane: $x+2y+3z+4=0$.

3(6pts) Sketch the surface of the equation $x+y^2+2z^2-1=0$, showing a few appropriate traces.

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4(18pts) Three points in the space are given: $P(0,1,2), Q(2,0,1), R(-1,-1,1)$.

(a) Find the equation of the plane containing the points.

(b) Find the area of the triangle with these points as its vertexes.

(c) Find the parametric equations of the line through point P and perpendicular to the plane.

5(7pts) The velocity of a moving particle is given as $\vec{v}(t) = \langle t, 2t, e^{2t} \rangle$. Find its position $\vec{r}(t)$ if $\vec{r}(0) = \langle 0,1,1 \rangle$.

6(14pts) Find the limit if exists, or show it does not exist by the 2-path rule.

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{\cos(xy)}{1+x^2+y^2}$

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy}{2x^2+y^3}$

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7(15pts) Consider the curve given by $\vec{r}(t) = \langle t, \cosh t \rangle$.

(a) Find the unit tangent vector $\vec{T}(t)$.

$\sinh t = \frac{e^t - e^{-t}}{2}$, $\cosh t = \frac{e^t + e^{-t}}{2}$
$\tanh t = \frac{\sinh t}{\cosh t}$, $\coth t = \frac{\cosh t}{\sinh t}$
$\text{sech} t = \frac{1}{\cosh t}$, $\text{csch} t = \frac{1}{\sinh t}$
$\cosh^2 t - \sinh^2 t = 1$, $\tanh^2 t + \text{sech}^2 t = 1$
$\sinh' t = \cosh t$, $\cosh' t = \sinh t$
$\tanh' t = \text{sech}^2 t$, $\text{sech}' t = -\text{sech} t \tanh t$
$\coth' t = -\text{csch}^2 t$, $\text{csch}' t = -\text{csch} t \coth t$

(b) Find the unit principal normal vector $\vec{N}(t)$.

(c) Find the curvature.

8(6pts) Find the distance from the point $(1, 1, 2)$ to the line which goes through $(1, 0, 1)$ and $(3, 2, -1)$.

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9(18pts) At an instance the following are given for a particle in motion: The acceleration $\vec{a} = (0, 3, 4)$, the velocity $\vec{v} = (-1, 0, 1)$. Find the following: (*Hint*: use the relation $\vec{a} = a_T\vec{T} + a_N\vec{N}$.)

(a) The speed $\frac{ds}{dt}$ at the instance.

(b) The tangential component of the acceleration a_T at the instance.

(c) The normal component of the acceleration a_N at the instance.

(d) The trajectory's curvature κ at the instance.

(d) The principal normal unit vector $\vec{N}$ at the instance.

(e) The binormal unit vector $\vec{B}$ at the instance.

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2 Bonus Points: Calculus was invented in: (a) The 17th century. (b) The 18th century. (c) The 19th century. (d) The 20th century. (... *The End*)