Math 208 Summer II 2006 **Final Exam**

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20pts) (a) Find the directional derivative of function $z = x^2 + y$ at the point $(1, 0)$ in the direction of $\langle 2, 1\rangle$.

(b) Find the tangent plane of the function at the point $(1, 0, 1)$.

2(20pts) (a) It is given that y can be solved as a function of x, z from the equation $2xe^{xy} + xz^2 + yz = 3$ at the point $(1, 0, -1)$. Use implicit differentiation to find $\frac{\partial y}{\partial z}(1, -1)$.

(b) Find the limit or show it does not exist for $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^3 + y^3}$

3(15pts) Sketch the region of integration and change the order of the iterated integral for $\int_0^2 \int_0^{4-x^2} f(x, y) dy dx$.

4(15pts) Find all critical points of $f(x, y) = 2x^3 - 3x^2 + y^2 - 6y$ and classify them by the 2nd derivative test.

5(15pts) A rectangular box with a square base is constructed with materials for the top and base cost, in unit price per square foot, twice as much as for the four sides. Find the dimension of a box to minimize the cost if the box is required to have 16 cubic feet in volume. (Use Lagrange multiplier method.)

6(20pts) At an instance the following are given for a particle in motion: its velocity $\vec{v} = \vec{r}\,' = (-1, 0, 1)$ and acceleration $\vec{a} = \vec{r}\,'' = (0, 3, 4)$. Find the following: (*Hint*: use the relation $\vec{a} = a_T\vec{T} + a_N\vec{N}$.)

(a) The unit tangent vector $\vec{T}$ at the instance.

(b) The tangential component of the acceleration $\dfrac{d^2s}{dt^2}$ at the instance.

(c) The principal normal unit vector $\vec{N}$ at the instance.

7(15pts) Set up an integral for the mass of a wire C which has the shape of a curve $y = x^3$ between $-2 \leq x \leq 2$ and has a density function given by $\rho(x, y) = x^2 + y^2$. (Do not evaluate the integral.)

8(15pts) Use the spherical coordinate to set up and then evaluate the volume of a solid ball of radius a, $Q : x^2 + y^2 + z^2 \leq a^2$. (Show the work. The answer should be $\frac{4\pi a^3}{3}$.)

9(20pts) (a) Show that the force field $\vec{F}(x, y) = \langle 2xy, x^2 + 2y \rangle$ is conservative by finding a potential function $f(x, y)$.

(b) Find the work done by the force field of (a) above on an object moving from point $(1, 1)$ to $(2, 8)$ along the cubic curve $y = x^3$.

10(15pts) Set up an integral in polar coordinate for the mass of a surface lamina $S: z = 1 - x^2 - y^2,\ z \geq 0$ with a density function $\rho(x, y, z) = z(x^2 + y^2)$. (Do not evaluate the integral.)

11(15pts) Let Q be the solid sphere $x^2 + y^2 + z^2 \leq 9$. Use the Divergence Theorem to find the flux of a vector field $\vec{F} = \langle z + x, x + y, y + z \rangle$.

12(15pts) Let C be the triangle with vertexes $(1, 0, 0), (0, 2, 0), (0, 0, 2)$ on the plane $2x + y + z = 2$, going counterclockwise when looking down. Use Stoke's Theorem to find the line integral $\displaystyle\oint_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F} = \langle z, x + y \sin y, y \rangle$.

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2 Bonus Points: The flux of a vector field through a surface is: (a) a number, (b) a vector, (c) a surface, (d) all above.