Math 208 Summer II 2005 Final Exam

August 11, 2005 **NAME** ______________________________

Show all work; guessing is not enough. 100 points possible (counting for 200 points in the course). Please identify answers clearly.

1. Let $\mathbf{r} = \langle t, \frac{t^2}{2}, 0\rangle$ describe the location at time t of an object moving in space.

   (a) (*6 pts*) Find the velocity and acceleration vectors when $t = 1$.

   (b) (*10 pts*) Find the unit tangent $\mathbf{T}$ and principal unit normal $\mathbf{N}$ at $t = 1$.

   (c) (*6 pts*) Find the radius of curvature at $t = 1$.

2. (*7 pts*) Find the projection (vector) of $\langle 1, 2, -3\rangle$ onto $\langle 0, 2, 1\rangle$.

3. (*8 pts*) Find the local extrema of $f(x, y) = x^2 - 2x + y^2 - 6y$, identifying them by means of the Second Derivative Test.

4. (*10 pts*) (a) Find the equation of the plane tangent to the surface $(x-1)^2 + y^2 + z^2 = 9$ at $(3, 1, 2)$, and (b) find the equations of the line through that point and normal to the surface.

5. (*8 pts*) Using a little calculus and a little symmetry, find the center of mass of a triangle with constant density $\rho = 1$ with vertices at $(0, 0), (0, 1)$, and $(1, 0)$.

6. (*7 pts*) Find the directional derivative of $f(x, y) = xy$ in the direction $\langle \frac{4}{5}, \frac{3}{5} \rangle$ at the point $(x, y) = (1, 3)$.

7. (*8 pts*) Determine the volume between the paraboloids $z = x^2 + y^2$ and $z = 9 - 2x^2 - 2y^2$ using a triple integral in cylindrical coordinates.

8. (*8 pts*) Briefly sketch the vector field $\langle 1, 2x \rangle$ and determine whether it is conservative.

9. (*7 pts*) A wire in space in the shape of the parabola $y = x^2$ from $x = \sqrt{2}$ to $x = \sqrt{6}$, $z = 0$, has mass density $\rho(x, y, z) = x$ per unit length. Find its mass using the arc length integral $\int_C \rho \, ds$ (and parameterizing, etc.).

10. (*7 pts*) Use Green's Theorem to calculate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle 3y, 5x \rangle$ over the curve C : $x^2 + y^2 = 4$, oriented counterclockwise.

11. (*7 pts*) Use the Divergence Theorem to calculate the flux $\iint_{\partial Q} \mathbf{F} \cdot \mathbf{n} \, dS$ of the field $\mathbf{F} = \langle x, y, 1 \rangle$ through the surface of the cube $0 \leq x \leq 3$, $0 \leq y \leq 3$, $0 \leq z \leq 3$, with $\mathbf{n}$ outward.