Math 208, Summer 2007, Exam 3

Show your work. Justify your conclusions.

[6] **1**. Let $w = f(x,y,z)$, $x = x(s,t)$, $y = y(s)$ and $z = z(t)$. Write out, *with correct derivative notation,* the chain rule expressions for w_s and w_t.

[4] **2**. Suppose that the function $f(u,v)$ satisfies Laplace's equation

$$f_{uu} + f_{vv} = 0.$$

Set

$$u = \frac{x^2 - y^2}{2}, \quad v = xy \quad \text{and} \quad w(x,y) = f(u(x,y), v(x,y)).$$

Show that w also satisfies Laplace's equation:

$$w_{xx} + w_{yy} = 0.$$

3. Let $z = f(x,y) = x^2/(1+y)$.

[6] **a**. Compute the linear approximation to f at $(2,1)$.

[4] **b**. Use the differential to approximate $f(2.02, .96) - f(2,1)$.

4. Let $g(x,y,z) = xy + (1/z)$.

[6] **a**. Find the derivative of g at $(2,1,1)$ in the direction $\vec{v} = \langle 1, 0, -1 \rangle$.

[4] **b**. In which direction is g increasing most rapidly at the point $(2,1,1)$? What is the derivative of g at $(2,1,1)$ in this direction?

[4] **5**. Find a vector normal to the surface $x^3 + xz + y^2 + z^2 = 4$ at the point $(1,-1,1)$.

6. Let $f(x,y) = 2x^2 - y^3 - 2xy$.

[4] **a**. Find the critical points of f.

[4] **b**. Classify the critical points. (Local maximum, local minimum or saddle point?)

[8] **7**. Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x,y) = e^{2x+y}$ subject to the constraint $x^2 + y^2 = 5$.