Exam 4 Solutions

1. The volume is

$$\iint_R (x+4)\,dA = \int_{-4}^{1}\int_{3x}^{4-x^2} (x+4)\,dy\,dx$$
$$= \int_{-4}^{1} (x+4)(4-x^2-3x)\,dx$$
$$\approx 52.083\,.$$

2a. With the order of integration $dx\,dy$,

$$\iint_R f(x,y)\,dA = \int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\,dx\,dy + \int_1^4 \int_{y-2}^{\sqrt{y}} f(x,y)\,dx\,dy.$$

2b. Reversing the order of integration, we get

$$\iint_R f(x,y)\,dA = \int_{-1}^{2}\int_{x^2}^{x+2} f(x,y)\,dy\,dx.$$

3. In polar coordinates,

$$\int_0^6 \int_0^y x\,dx\,dy = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{6\csc\theta} r^2\,dr\,d\theta = 36.$$

4a. The paraboloids intersect over the disk R of radius 2 centered at the origin in the xy-plane. Hence the volume of D is

$$\iiint_D dV = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{8-x^2-y^2} dz\,dy\,dx$$
$$= 4\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{x^2+y^2}^{8-x^2-y^2} dz\,dy\,dx.$$

I used the symmetry of the region D to get the second triple iterated integral.

4b. With order of integration $dx\,dy\,dz$,

$$\iiint_D dV = \int_0^4 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} dx\,dy\,dz + \int_4^8 \int_{-\sqrt{8-z^2}}^{\sqrt{8-z^2}} \int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} dx\,dy\,dz.$$

As in part (a) you can use the symmetry of D to simplify your expression:

$$\iiint_D dV = 4\int_0^4 \int_0^{\sqrt{z}} \int_0^{\sqrt{z-y^2}} dx\,dy\,dz + 4\int_4^8 \int_0^{\sqrt{8-z^2}} \int_0^{\sqrt{8-z-y^2}} dx\,dy\,dz.$$

5. In cylindrcal coordinates,

$$\iiint_D (x^2 + y^2 - z)\, dV = \int_0^{2\pi} \int_0^{2} \int_{r^2}^{8-r^2} (r^2 - z) r \, dz \, dr \, d\theta.$$

6. For $0 \leq \varphi \leq \pi/3$, ρ goes from 0 to $\sec\varphi$. And for $\pi/3 \leq \varphi \leq \pi/2$, ρ goes from 0 to 2. Thus the volume of D is

$$\iiint_D dV = \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^{\sec\varphi} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta + \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^{2} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta.$$