MATH 208 **Name:** ____________________

Exam # 3

This test is worth 100 points. Show all of your work to receive full and partial credit, making sure that all work and answers are legible. Books, notes, formula sheets, etc. are not allowed. Throughout this exam, whenever you use a theorem, you must name the theorem for full credit.

1. (a) (14 points) $\vec{F} = (8x^3 + 4y)\vec{i} + (4x + 2y)\vec{j}$ is a gradient field. Find a potential function for $\vec{F}$.

   (b) (6 points) Find $\int_C \vec{F} \cdot \vec{dr}$ where $\vec{F}$ is the field from part a) and C is the straight line segment from $(0, 1)$ to $(1, 2)$.

2. (12 points) Use the curl test to determine if $\vec{F} = (y - 4xy)\vec{i} + (x - 2x^2)\vec{j}$ is path independent.

3. (16 points) Find the flux of $\vec{F} = (x - 2xy)\vec{i} + (6xy - y + 3z)\vec{j} - 6xz\vec{k}$ through the box of height 2, length 5, and width 3, sitting on the xy-plane with one corner at the origin and the long edge extending in the x direction. The box is oriented outward. If you use a theorem, name the theorem.

4. (12 points) $\vec{F} = (4y^2 - 2z)\vec{i} + e^{xy}\vec{j} + 3xyz\vec{k}$. Find curl$\vec{F}$.

5. (10 points) Find a parametrization for the cylinder of radius 4 centered on the x-axis between $x = -3$ and $x = 3$.

6. (16 points) Set up, but **do not evaluate**, the double integral to find the flux of $\vec{F} = y\vec{i} - 2\cos(x)\vec{j} + xz\vec{k}$ through the surface S where S is parameterized by $\vec{r}(s,t) = (t^2-1)\vec{i} + 5s\vec{j} + st^2\vec{k}$, $0 \leq s \leq 5, -2 \leq t \leq 3$, and is oriented upward.

7. (8 points) Indicate, by writing TRUE or FALSE if each of the following statements is correct as written. (You do not need to give a reason for your answer.)
   (a) To apply Green's Theorem, the vector field must be path independent.
   (b) The flux of the vector field $\vec{F} = x\vec{j}$ through the plane $y = 0$, with $0 \leq x \leq 1, 0 \leq z \leq 1$ oriented in the positive y direction is positive.
   (c) The circulation density, $\text{circ}_{\vec{n}}\vec{F}$ is a scalar.
   (d) If $\vec{F}$ is a vector field in 3-space, then $\text{curl}\vec{F}$ is also a vector field.

8. (6 points) For each of the questions below, determine if one of the theorems learned in class can be used to answer the question. If so, write the name of the theorem. If not, write "no theorem can be used." You do not need to answer the questions.
   (a) Find $\int_C \vec{F} \cdot dr$, where $\vec{F} = 4\vec{i} - 2\vec{j}$ and C is the line from $(0,4)$ to $(3,8)$.
   (b) Find the circulation of $\vec{F} = \dfrac{-y\vec{i} + x\vec{j}}{x^2+y^2}$ around the circle $x^2 + y^2 = 4$ oriented counterclockwise.