**MATH 208** **Name:** ____________________

**Spring 2015** **Exam # 2**

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This test is worth 100 points. Show all of your work to receive full and partial credit, making sure that all work and answers are legible. Books, notes, formula sheets, etc. are not allowed.

1. (14 points) The function $f(x, y) = x^3 - 3x + y^3 - 3y$ has critical points at $(1, 1), (-1, 1), (1, -1)$ and $(-1, -1)$. Classify each of these critical points as a local maximum, local minimum, or saddle point. (Note: you do not need to verify that these are critical points).

2. (a) (16 points) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = (x - 2)^2 + (y + 1)^2$ subject to the constraint $x^2 + y^2 = 20$.

   (b) (7 points) Find the maximum and minimum values of $f(x, y)$ subject to the constraint $x^2 + y^2 \leq 20$.

3. Consider the iterated integral: $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} 4(x^2 + y^2) dy dx$.

   (a) (10 points) Sketch the region of integration.

   (b) (10 points) Convert the integral to polar coordinates.

4. (18 points) Set up a triple integral in Cartesian coordinates to calculate the volume of the region between the planes $4x+3y-z = -7$ and $z = 2$, and above the triangle with vertices $(0, 0, 0)$, $(5, 0, 0)$, and $(0, 3, 0)$. You do not need to evaluate the integral.

5. (4 points) Circle your answer. The integral: $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{16} \rho^2 \sin\phi d\rho d\phi d\theta$ calculates:

   (A) The mass of an object with density $\delta = \rho^2 \sin\phi$, where the object is the region inside the sphere of radius 4, and under the cone $z = \sqrt{x^2 + y^2}$.

   (B) The mass of an object with density $\delta = \rho^2 \sin\phi$, where the object is the region inside the sphere of radius 4, and above the cone $z = \sqrt{x^2 + y^2}$.

   (C) The volume of the region inside the sphere of radius 4, and under the cone $z = \sqrt{x^2 + y^2}$.

   (D) The volume of the region inside the sphere of radius 4, and above the cone $z = \sqrt{x^2 + y^2}$.

6. (9 points) Find a parametrization for the line segment from $(3, 4, 1)$ to $(1, -3, 5)$.

7. Consider the vector field $\vec{F} = x\vec{j}$.

   (a) (2 points) Sketch the vector field.

   (b) (2 points) Sketch the flow of $\vec{F}$.

   (c) (8 points) Consider the parametrization $x(t) = a, y(t) = at$. Is $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ a flow line of $\vec{F}$? Show why or why not.

Have a good break!