Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(16pts) (8pts each) (a) Evaluate the integral \( \int \frac{x^2 + x + 2}{x^3 + 2x^2} \, dx \)

(b) Find a correct form of partial fraction for \( \frac{2x^2 + 3x + 3}{(x + 1)^3(x^2 + 2x + 3)^2} \). Do not solve for the constants.
2(16pts) (8pts each) Determine by definition whether the improper integrals converge. Find the value of any convergent integral. Make sure to show all details.

(a) \[
\int_{2}^{3} \frac{2x}{\sqrt{x^2 - 4}} \, dx
\]

(b) \[
\int_{0}^{\infty} \frac{\tan^{-1} x}{1 + x^2} \, dx
\]

3(16pts) (8pts each) Use comparison tests to determine whether or not the improper integrals converge.

(a) \[
\int_{1}^{\infty} \frac{1}{1 + x^3} \, dx
\]

(b) \[
\int_{1}^{2} \frac{2 + \sin x^2}{(x - 1)^2} \, dx
\]
4 (18pts) (9pts each) Determine if the series converge. Find the sum of any convergent series. Make sure to include sufficient details.

(a) \[ \sum_{k=2}^{\infty} (-1)^k \frac{2^{k+1}}{3^k} \]

(b) \[ \sum_{k=0}^{\infty} \frac{k \cos(1/k^2)}{2k + 1} \]

5 (10pts) Use the Integral Test to determine if the series \[ \sum_{k=1}^{\infty} ke^{-k} \] converges. Make sure to verify all the conditions of the test.
(a) Determine whether the sequence \( \lim_{n \to \infty} \frac{(-1)^n n^2 + 1}{2n^2 + 2n + 1} \) converges. If it does, find the limit.

(b) Demonstrate that the sequence \( a_n = \frac{n + 2}{n + 1} \) is monotone decreasing and find its limit \( \lim_{n \to \infty} a_n \).

(c) Numerically approximate the infinite sum \( \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)2^k} \) to 4 decimal places. What is the minimum number of terms that is needed to obtain the required accuracy?

2 Bonus Points: Pierre Simon Laplace worked on (a) a new calendar for Napoleon, (b) improper integrals to develop the Laplace transform, (c) a French vineyard as a slave laborer. (Circle all that are true)