1 (16 pts) Evaluate the integrals (a) \[ \int \frac{2x}{\sqrt{5 + 2x + x^2}} \, dx \]

(b) \[ \int_1^4 \frac{x^3 - 1}{\sqrt{x}} \, dx \]
2(18pts) (a) Find the derivative of the function \( y = f(x) = x^3 + x + 1 \) and use it to show \( f \) is invertible.

(b) Show that the point \((1, 3)\) is on the graph of the function \( f \), and find an equation of the tangent line at the point \((3, 1)\) for the inverse function \( y = f^{-1}(x) \).

(c) An invertible function is as shown. Sketch the graph of its inverse.

3(16pts) (a) Derive and simplify \( \frac{d}{dx} \cot^{-1}x \) for which \( \cot^{-1}x \) is defined as the inverse of cotangent function \( \cot x = \frac{\cos x}{\sin x} \) from the interval of \((0, \pi)\) to \((-\infty, \infty)\). Note that \( \cot' x = -\csc^2 x \).

(b) Suppose a batch of bacteria initially has 100 cells. After 2 hours, the population has increase to 400. Assume that the population grows exponentially. What will the population be after 8 hours?
4(20pts) Evaluate the integrals by the method of integration by parts.

(a) $\int x \ln x \, dx$

(b) $\int x^2 \sin x \, dx$

5(10pts) Evaluate the integral $\int \frac{1}{(x-2)^2 \sqrt{x}} \, dx$, using the following formulas

$$\int \frac{1}{u^n \sqrt{a + bu}} \, du = \frac{-1}{a(n-1)} \frac{\sqrt{a + bu}}{u^{n-1}} - \frac{(2n-3)b}{2a(n-1)} \int \frac{1}{u^{n-1} \sqrt{a + bu}} \, du$$

$$\int \frac{1}{u \sqrt{a + bu}} \, du = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C$$

(Continue on Next Page ...)
Evaluate the trigonometric integrals

(a) \( \int \sin^3 x \, dx \)

(b) \( \int \tan x \sec^4 x \, dx \)

2 Bonus Points: An effective trigonometric substitution for the integral \( \int \sqrt{4 + x^2} \, dx \) should be \( x = \) _________.

(... The End)