Math 107-Sec 510, Summer '03 **Exam 2** **Score:** ____________

Name: ______________________________ **TA's Name:** ________________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(16pts) Evaluate the integrals (a) $\displaystyle\int \frac{2x}{\sqrt{5+2x+x^2}}dx$

(b) $\displaystyle\int_1^4 \frac{x^3-1}{\sqrt{x}}dx$

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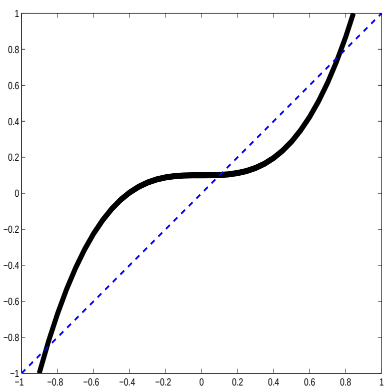
2(18pts) (a) Find the derivative of the function $y = f(x) = x^3 + x + 1$ and use it to show f is invertible.

(b) Show that the point $(1, 3)$ is on the graph of the function f, and find an equation of the tangent line at the point $(3, 1)$ for the inverse function $y = f^{-1}(x)$.

(c) An invertible function is as shown. Sketch the graph of its inverse.



3(16pts) (a) Derive and simplify $\frac{d}{dx}\cot^{-1} x$ for which $\cot^{-1} x$ is defined as the inverse of cotangent function $\cot x = \frac{\cos x}{\sin x}$ from the interval of $(0, \pi)$ to $(-\infty, \infty)$. Note that $\cot' x = -\csc^2 x$.

(b) Suppose a batch of bacteria initially has 100 cells. After 2 hours, the population has increase to 400. Assume that the population grows exponentially. What will the population be after 8 hours?

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4(20pts) Evaluate the integrals by the method of integration by parts.

(a) $\int x \ln x dx$

(b) $\int x^2 \sin x dx$

5(10pts) Evaluate the integral $\int \frac{1}{(x-2)^2\sqrt{x}} dx$, using the following formulas

$$\int \frac{1}{u^n\sqrt{a+bu}} du = \frac{-1}{a(n-1)} \frac{\sqrt{a+bu}}{u^{n-1}} - \frac{(2n-3)b}{2a(n-1)} \int \frac{1}{u^{n-1}\sqrt{a+bu}} du$$

$$\int \frac{1}{u\sqrt{a+bu}} du = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}} \right| + C$$

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6(20pts) Evaluate the trigonometric integrals

(a) $\int \sin^3 x dx$

(b) $\int \tan x \sec^4 x dx$

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2 Bonus Points: An effective trigonometric substitution for the integral $\int \sqrt{4+x^2}dx$ should be $x =$ ____________.

(... *The End*)