Math 107-Sec 35X, Spring '05 **Test 3** **Score:** ____________

Name: ______________________________ **TA's Name:** ________________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20pts) Find the interval in which the following power series converges absolutely. Also find the radius of convergence and discuss in details the convergence/divergence of the series at the end points of the interval of convergence.

$$\sum_{k=0}^{\infty} \frac{(x-1)^k}{\sqrt{4k+1}}$$

(*Continue on Next Page ...*)

2(28pts) (7pts each) (a) Use the Taylor series for $\frac{1}{1-x}$ at $x = 0$ to represent the integral $\displaystyle\int_0^1 \frac{x}{1+x^4}$ as an infinite series.

(b) If we want to approximate the series to 3 decimal place accuracy using a nth partial sum, what is the n, the number of terms used for the approximation? Explain your reasoning for the error estimate.

(c) Using a known Taylor series to find the exact value of

$$\sum_{k=0}^{\infty}(-1)^k \frac{2}{k!}$$

(d) Use the Taylor series of function $f(x) = \dfrac{x}{1-2x^3}$ to find $f^{(100)}(0)$.

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3(28pts) (7pts each) (a) Find the tangent line of the curve $x(t) = \cos 2t, y(t) = \sin 3t$ at the point corresponding to $t = \pi/2$.

(b) Find the intersection points of the line through $(-1, -2)$, $(3, 6)$ and another curve $x(t) = t + 1,\ y(t) = t^2 - 1$. (*Hint*: parameterize the line first.)

(c) Find a parametrization of the ellipse $\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$, going clockwise.

(d) Find the Taylor polynomial $P_2(x)$ of $g(x) = \dfrac{1}{\sqrt{x}}$ at point $x = 1$.

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4(24pts) (8pts each) Determine whether the following series converge absolutely, converge conditionally, or diverge. **Make sure to verify all conditions of any test used.**

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{2^k}{(2k+1)!}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k(2k-1)}{k^{2.01}+1}$

(c) $\displaystyle\sum_{k=1}^{\infty} ke^{-k^2}$

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2 Bonus Points: Albert Einstein published his theory on special relativity: (a) 10 years ago, (b) 100 years ago, (c) 90 years ago, (d) 50 years ago. (... *The End*)