Math 107 Project: Houdini's Great Escape

Due Date: April 14, 2005

Guidelines: Keep in mind that part of your grade is based on the quality of your written presentation. The paper you turn in should have a mix of equations, formulas and prose. Graphs may be copied from your calculator, but should be clearly labeled. Use complete sentences, good grammar as well as correct punctuation and spelling. Try to justify your answers as much as possible. You should aim to write your report in such a way that it can be read and understood by another calculus student.

Harry Houdini was a famous escape artist. In this project we relive a trick of his that not only challenged his skill and bravery, but his mathematical prowess as well.

Houdini had his feet shackled to the top of a concrete block which was placed on the bottom of a water tank made of glass and shaped like a giant laboratory flask (with circular cross-sections). The cross-sectional radius of the tank, measured in feet, was given by the function

$$r(z) = \frac{10}{\sqrt{z+1}}$$

where z is the height from the bottom of the tank. The tank was then filled with water at a steady rate of 22π cubic feet per minute. Houdini's task was to escape the shackles before he was drowned by the rising water.

Now Houdini knew it would take him exactly 10 minutes to escape the shackles. For maximum effect on his audience, he wanted to time his escape so it was completed precisely at the moment the water level reached the top of his head. Houdini was exactly 6 feet tall. In the design of the apparatus, he was allowed to adjust only one thing: the height of the concrete block he stood on.

A. Express the volume of the water in the tank as a function of the height of the water from the bottom. (For mathematical simplicty, ignore Houdini's volume and the volume of the concrete block.)

B. If everything is timed perfectly, what should the volume of the water in the tank be when the water reaches the top of Houdini's head?

C. Using your answers from **A** and **B**, what should be the height of the block?

D. Now suppose Houdini tries his trick in a tank with radius given by

$$r(z) = \frac{11}{\sqrt{z+\ln(z+19)}}$$

and a concrete block of height 4 feet. Assume Houdini can hold his breath for no more than two minutes. What happens? Does Houdini escape? Remember to justify your answer.

E. Let $r(z)$ be as in part **D** and let $h(t)$ be the height of the water at time t. Houdini was an expert at calculus and in order to check the progress of his escape moment by moment, he derived a formula for $\frac{dh}{dt}$ in terms of h itself. Derive this formula. (Hint: First find $\frac{dV}{dh}$.) How fast is the water level changing when the water just reaches the top of his head?