Appendix 2: Cycloids

Description of a Cycloid

A cycloid is the curve followed by a point S on the circumference of a circle as the circle is rolled along a horizontal line (see Figure 1). A useful way to represent a cycloid, with a cusp at $(0, 0)$, is by

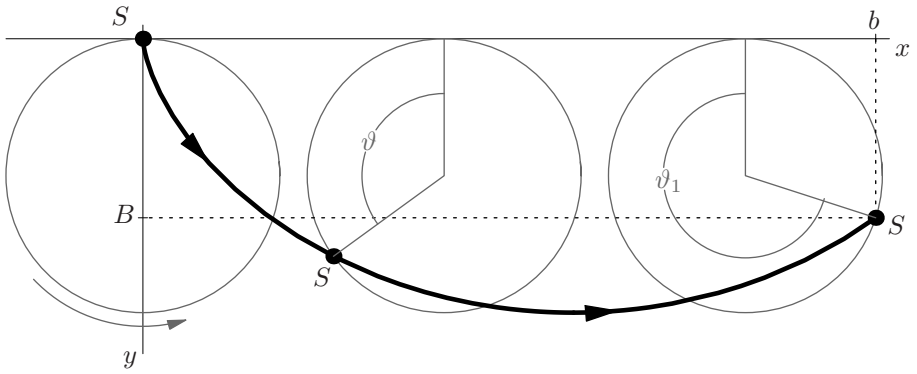

Figure 1: Generation of a cycloid

the parametric equations

$$x(\theta) = R(\theta - \sin\theta) \text{ and } y(\theta) = R(1 - \cos\theta) \quad \text{for all } \theta \in [0, \theta_1],$$

where $\theta_1 \in (0, 2\pi]$ and $R \in (0, +\infty)$. The quantity R is the radius of the circle that is being rolled. To find a cycloid that passes through the points $(0, 0)$ and (b, B), the values for R and θ_1 should be chosen so that $x(\theta_1) = b$ and $y(\theta_1) = B$.