Math 107H Fall '09 **Exam 2** **Score:** _____

Name: _____ **TA's Name:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(10pts) Find the Taylor polynomial $P_2(x)$ and the remainder $R_2(x)$ of $f(x) = \sqrt{x}$ at point $x = 4$.

2(10pts) Find the sum $\sum_{n=1}^{\infty}(\arctan(n) - \arctan(n+1))$.

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3(10pts) Use the Taylor series for $\sin x$ at $x = 0$ to represent the integral $I = \int_0^1 \frac{\sin(x^2)}{x} dx$ as an infinite series.

4(10pts) To use the partial sum to approximate the infinite series $\sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2+1}$ to the first 3 decimal places (i.e. with an error no greater than 10^{-5}), how many terms do we need for the approximation? Show all work.

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5(25pts) (a) Use a comparison test to determine if $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2}{n^3+3n+3}$ converges.

(b) Use the Integral Test to determine if $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$ converges.

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6(25pts) Find the following for the power series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2n+1}$: (a) center and radius of convergence; (b) interval of absolute convergence; (c) points of conditional convergence if any, (d) interval of convergence; (e) intervals of divergence,

7(10pts) Use the Taylor series $\arctan x = \sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)$ to find $f^{(100)}(0)$ if $f(x) = \dfrac{\arctan x}{x}$ for $x \neq 0$ and $f(0) = 1$.

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2 Bonus Points: True or False: The official language of Canada is Canadian. _____ (... *The End*)