Math 107 Fall 2009 **FINAL EXAM** Name ____________________

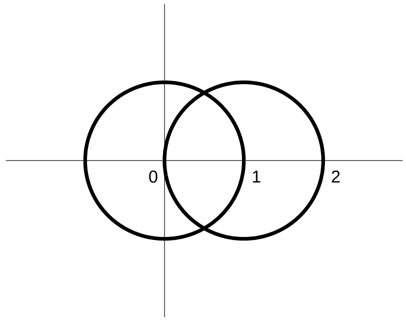
Student ID Number: ____________________

**INSTRUCTIONS:**

- Turn off all communication devices.
- This exam should have 10 pages of questions plus this cover sheet; please verify that it does.
- A note card of 3″ by 5″ is allowed.
- Calculators are allowed (except for TI-89's, TI-92's and similar models with symbolic algebra capability). **An answer will only be counted if it is supported by all the work necessary.**
- Do not spend too long on any one problem; note the point value of the problem when deciding how much time to spend! You do not need to work the problems in the order they appear.
- Work each problem completely and clearly in the space provided. Show all your work for full credit. If you use the back of exam pages for scratch work, make note of that in the space provided for the problem.
- Simplify as much as possible, except as noted. For example, write $\sqrt{2}/2$ instead of $\cos(\pi/4)$.
- Give exact answers only, except as noted. For example, use $\pi$ instead of 3.1415 if $\pi$ is the answer.
- Cheating in any shape or form will result in an automatic F for the course grade, and possible expulsion from the university.

| Page | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| 6 | 20 | |
| 7 | 20 | |
| 8 | 20 | |
| 9 | 20 | |
| 10 | 20 | |
| Total | 200 | |

1. Consider the region outside the unit circle and inside the circle $r = 2\cos\theta$:




(a) (10 points) Set up, but **DO NOT EVALUATE**, an integral for the area of the region.

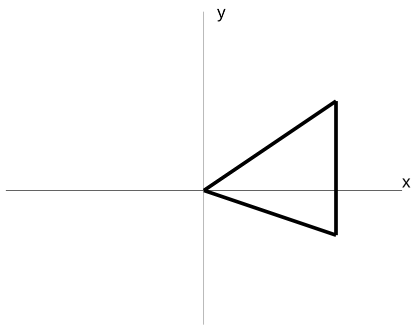
(b) (10 points) Find an equation for the tangent line to the circle $r = 2\cos\theta$ at the point where the two curves intersect in the first quadrant.

2. **Evaluate** the following improper integrals. If an integral diverges, indicate so and provide calculations showing that it diverges.

   (a) (10 points) $\displaystyle\int_1^2 \frac{x^2}{\sqrt{8-x^3}}dx$

   (b) (10 points) $\displaystyle\int_1^\infty \frac{2}{x^3+x}dx$. (Suggestion: Use partial fraction to find the indefinite integral.)

3. Consider the region bounded by $y = x,\ y = -x/2,\ x = 2$:




(a) (10 points) Set up, but **DO NOT EVALUATE**, an integral for the volume of the solid generated by revolving the region around the $y$-axis.

(b) (10 points) Set up, but **DO NOT EVALUATE**, an integral for the volume of the solid generated by revolving the region around the line $y = -1$.

4. For each of the following series, determine if it absolute converges, conditionally converges, or diverges. Justify your answer by verifying the conditions of the tests used.

   (a) (7 points) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

   (b) (5 points) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \arctan k$

   (c) (8 points) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{\sqrt{k}+1}{k^2+k+1}.$

5. Consider this power series: $\sum_{k=0}^{\infty} \frac{1}{2^k \sqrt{k+1}} (x+2)^k$.

   (a) (8 points) Find the center and radius of convergence.

   (b) (8 points) Find the interval of convergence.

   (c) (4 points) Find the interval of absolute convergence.

6. (20 points) A cylindrical fuel tank of 10 feet in height and 30 feet in diameter is full with gasoline weighing 46 lb per cubic foot. Find the work done to pump half of the fuel over the top of the tank.

7. (a) (12 points) For the function $f(x) = \sqrt{x}$, find the Taylor polynomial $P_2(x)$ of order 2 centered at $x = 4$, and its remainder $R_2(x)$.

(b) (8 points) Use the power series for $\dfrac{1}{1-x}$ to derive the power series for $\dfrac{1}{(1+2x)^2}$.

8. An object's trajectory is given by the position vector

$$\vec{r}(t) =< 2t + 1, t^2 + 2, -49t^2 + 980t >$$

with $t$ in seconds and distance in feet. The first two components represent the object's displacement from the origin due East and North respectively, and the third component represents its altitude.

(a) (5 points) What is the vertical component of the velocity of the object when $t = 5$ seconds?

(b) (5 points) At $t = 5$, is the height of the object increasing, decreasing or neither?

(c) (5 points) What is the object's speed at $t = 5$ seconds?

(d) (5 points) Set up, but **DO NOT EVALUATE**, an integral for the arc length of the object's path from $t = 0$ to $t = 5$ seconds.

9. The following problems refer to the vector $\vec{u} = 5\vec{i} - 12\vec{k}$, and the point $P = (1, -2, 0)$ and $Q = (3, 1, 2)$.

   (a) (5 points) Find the vector with 4 units of length but opposite in direction to $\vec{u}$.

   (b) (8 points) Find the point $R$ satisfying $\overrightarrow{PR} = 13\vec{u} + \overrightarrow{PQ}$.

   (c) (7 points) Find the angle between $\overrightarrow{PQ}$ and $\vec{u}$.

10. Consider the space curve given by

$$\vec{r}(t) = \cos t\vec{i} + 2\sin t\vec{j} + 3t\vec{k}.$$

(a) (10 points) Find a vector equation for the line tangent to the curve at the point when $t = \pi$.

(b) (10 points) Find the point at which the tangent line and the plane, $x + y + z = -1$, intersect.

(*End of Exam*)