

Name: _____

TA's Name: _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(10pts) (a) A curve is given in polar coordinate $r = 4 \tan \theta \sec \theta$. Transform it in the Cartesian coordinate.

(b) Express the hyperbola $xy = 1$ in polar coordinate.

2(10pts) Find the exact value for the length of the curve: $r = \sqrt{1 + \cos(2\theta)}$, $-\pi/2 < \theta < \pi/2$.

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3(10pts) Sketch by hand the cardioid with one inside loop: $r = a(\frac{1}{2} + \sin \theta)$. Make sure to include work on intervals in which r is positive or negative, increasing or decreasing.

4(10pts) Find the solution to the differential equation $\frac{d\vec{r}}{dt} = 3(t+1)^{1/2}\vec{i} + \frac{1}{t+1}\vec{k}$ with the initial condition $\vec{r}(0) = \vec{j}$.

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5(10pts) (a) For this four-leaved rose $r = \cos(2\theta)$, find the point in the Cartesian coordinate corresponding to $\theta = \pi/6$.

(b) Find the slope of the tangent line to the curve at that point.

6(10pts) (a) Find the intersection of the circle $r = 3 \sin \theta$ and the cardioid $r = 1 + \sin \theta$.

(b) Set up an integral for the area of the region inside the circle and outside the cardioid. Give a rough sketch of the region. (Do not evaluate the integral.)

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7(10pts) Given the vectors $\vec{u} = \langle 1, 2, 3 \rangle$, $\vec{v} = \langle 3, 2, 1 \rangle$.

(a) Find the angle between \vec{u} and \vec{v} .

(b) Find the unit vector of \vec{v} .

(c) Find the projection vector of \vec{u} along \vec{v} .

8(10pts) The trajectory of a particle moving in space is given as $\vec{r}(t) = \sin(t^2)\vec{i} + \ln(t^2 + 1)\vec{j} + t\vec{k}$. Find (i) its position, (ii) velocity, and (iii) acceleration at the point $\vec{r}(0)$.

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9(10pts) Find a parametric equation for the line through these points: $A(3, 4, -5)$, $B(-1, 0, -3)$.

10(10pts) A projectile is fired at a speed of 500 m/sec at an angle of 60° . Find the range of the projectile.

2 Bonus Points: Darwin's first book on evolution by natural selection was published 150 years ago in the month of: (a) May, (b) July, (c) September, or (d) November. (... *The End*)