Math 107-Sec 25X, Fall '03 **Test 3** **Score:** ____________

Name: ______________________________ **TA's Name:** ________________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(18pts) (6pts each) (a) Use the Taylor series for e^x at $x = 0$ to represent the integral $\int_0^1 e^{-x^2}$ as an infinite series.

(b) If we use the partial sum of the first 3 nonzero terms to approximate the integral, determine to which decimal place the approximation agrees with the integral. Explain your reasoning for the error estimate.

(c) Using a known Taylor series to find the exact value of

$$\sum_{k=1}^{\infty} (-1)^k \frac{\pi^{2k}}{(2k)!}$$

(*Continue on Next Page ...*)

2(14pts) Find the interval in which the following power series converges absolutely. Also find the radius of convergence and discuss in details the convergence/divergence of the series at the end points of the interval of convergence.

$$\sum_{k=0}^{\infty} \frac{x^k}{k+1}$$

3(16pts) (8pts each) (a) Use the Taylor series $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, $|x| < \infty$ to find the Taylor series of $f(x) = xe^{-x^2}$. Use it to find $f^{(100)}(0)$.

(b) Find the Taylor polynomial $P_3(x)$ of $g(x) = \frac{1}{\sqrt{x+1}}$ at point $x = 0$.

(*Continue on Next Page ...*)

4(20pts) (7pts each for (a,b)) Determine whether the following series converge absolutely, converge conditionally, or diverge. **Make sure to verify all conditions of any test used.**

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k+1}}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k(\ln k)^2}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{(2k^2-1)}$

(*Continue on Next Page ...*)

5(16pts) (8pts each) Consider the parameterized curve $x(t) = 2t^2, y(t) = t^2 + 1,\ 0 \leq t \leq 1$.

(a) Find all the points on the curve at which the slope of the tangent line equals 1.

(b) Set up an integral for the area of the surface formed by revolving the curve about $x = 2$. (**Do not evaluate the integral.**)

6(16pts) (8pts each) Find parametric equations describing the curves.

(a) The line segment from $(-2, 4)$ to $(6, 1)$.

(b) The circle of radius 2 centered at $(2, 1)$, drawn clockwise.

---

2 Bonus Points: Colin Maclaurin is (a) a French mathematician, (b) a Scottish mathematician, (c) discovered the Integral Test, (d) discovered the Maclaurin Test. (Circle all that are true) (... *The End*)