Math 107-Sec 25X Fall '03 **Exam 2** **Score:** _____

Name: _____ **TA's Name:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(18pts) (9pts each) (a) Evaluate the integral by the method of integration by parts $\int xe^{2x}dx$

(b) Evaluate the integral by completing squares: $\int \frac{x}{x^2+2x+2}dx$

(*Continue on Next Page ...*)

2(18pts) (9pts each) (a) Use trigonometric substitution to evaluate $\int \sqrt{1+x^2}\,dx$. You may use the formulas:

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx \text{ and } \int \sec x\,dx = \ln|\sec x + \tan x| + c$$

(b) Evaluate the integral $\int \frac{2x+2}{x^2+2x}dx$ using partial fractions.

(*Continue on Next Page ...*)

3(16pts) (8pts each) Determine by definition whether the improper integrals converge. Find the value of any convergent integral. **Make sure to show all details.**

(a) $\displaystyle\int_1^2 \frac{x^2}{\sqrt{x^3-1}}\,dx$

(b) $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$

(*Continue on Next Page ...*)

4(16pts) (8pts each) Use comparison tests to determine whether or not the improper integral converge: $\int_1^\infty e^{-x^{3/2}}dx$

(b) Evaluate the trigonometric integral $\int \sin^2 x \cos^3 x \, dx$.

(*Continue on Next Page ...*)

5(16pts) (8pts each) (a) Find the limit of the sequence $\lim_{n\to\infty} \dfrac{n^2-2}{2n^2+n+2}$. **Make sure to include sufficient details.**

(b) Use the Squeeze Theorem to find the limit $\lim_{n\to\infty} \dfrac{\sqrt{n}+\sin n}{\sqrt{n}+2}$. **Make sure to include sufficient details.**

(*Continue on Next Page ...*)

6(16pts) (8pts each) Write down the first few terms in the series. Determine if the series converge. Find the sum of any convergent series. **Make sure to include sufficient details.**

(a) $\sum_{k=1}^{\infty}(-1)^{k-1}\frac{e^k}{3^{k+1}}$

(b) $\sum_{k=0}^{\infty}(-1)^k \cos\frac{1}{k^2}$

---

2 Bonus Points: The Taylor's Theorem is named after (a) Christian Taylor, (b) Charles Taylor, (c) Brook Taylor. (*... The End*)