Math 107 Fall 2003 **FINAL EXAM** **Name** _____

Dec. 16, 2003 **Student ID Number:** _____

Circle your instructor's name.

Instructor: Deng Haataja Rogge Thomas

Circle your TA's name, if applicable.

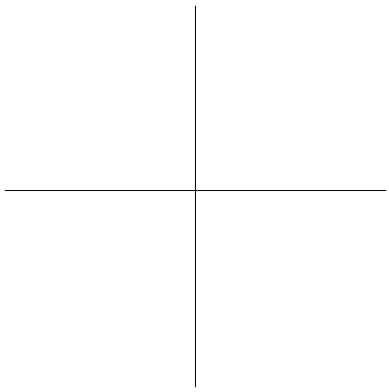
TA: Eubanks Evans Everson Harrington Higgins Miller

INSTRUCTIONS:

- This exam should have 6 pages of questions plus this cover sheet; please verify that it does.
- Work each problem completely and clearly in the space provided. Show all your work for full credit. If you use the back of exam pages for scratch work, make note of that in the space provided for the problem.
- Simplify as much as possible, except as noted. For example, write $\sqrt{2}/2$ instead of $\cos(\pi/4)$.
- Give exact answers only, except as noted. For example, use π instead of 3.1415 if π is the answer.
- Calculators are allowed (except TI-89's, TI-92's and similar models), but **an answer will only be counted if it is supported by all the work necessary to obtain that answer.**
- Do not spend too long on any one problem; note the point value of the problem when deciding how much time to spend! You do not need to work the problems in the order they appear.

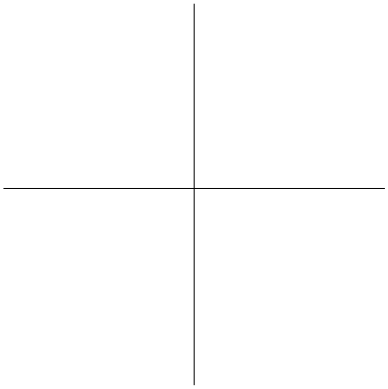
Question	Points	Score
1	14	
2	14	
3	14	
4	14	
5	12	
6	36	
7	20	
8	12	
9	8	
10	18	
11	8	
12	12	
13	18	
Total	200	

1. (14 points) Construct a definite integral to determine the area enclosed by the graphs of $x - y^2 = 1$ and $y = -x + 3$. **DO NOT EVALUATE THIS INTEGRAL.**



2. (14 points) Set up integral(s) to compute the volume of the solid formed by rotating the region bounded by $y = \frac{1}{2}x$, $y = x - 2$, and the x-axis about the line $x = -3$.

   (a) Sketch the region, carefully labeling all relevant quantities.



   (b) Set up integral(s) to compute the volume of this solid. **DO NOT EVALUATE THIS INTEGRAL.**

3. (14 points) A water tank in the shape of a right circular cylinder has a base radius of 5 feet and a height of 10 feet. Suppose that the tank contains 7 feet of water. Set up an integral to determine the work done in pumping all of the water out of the top of the tank. **DO NOT EVALUATE THIS INTEGRAL.**

4. (a) (7 points) Find the exact value of $\sin^{-1}(\sin(3\pi/4))$

   (b) (7 points) Find $\dfrac{d}{dx}\sin^{-1}(\tan x)$

5. (12 points) Determine whether the improper integral converges or diverges. If it converges, find the limit.

$$\int_0^1 \frac{x}{x^2-1}dx$$

6. (12 points each) Evaluate the integrals.

(a) $\int \cos^3 x \sin^2 x dx$

(b) $\int_1^e (\ln x)^2 dx$

(c) $\int \frac{x-4}{x^2+2x} dx$

7. (10 points each) Given the Taylor series centered at 0 (the Maclaurin series) for $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$, for all x.

   (a) Use this series to estimate e accurate to three decimal places. How many terms are required? You must show your work.

   (b) Use the first three terms of the Maclaurin series for e^x to estimate $\displaystyle\int_0^1 e^{-x^2}\,dx$. You must show your work.

8. (12 points) A rubber ball, when dropped on concrete, rebounds 90 percent of the distance it falls. If it is dropped from a height of 6 feet, how far does it travel, both up and down, before coming to rest?

9. (8 points) Find the radius of convergence of this power series: $\sum_{k=0}^{\infty} \frac{k^2}{3^k} x^k$

10. (9 points each) Determine if each of the following series converges absolutely, converges conditionally, or diverges. Show enough work to justify your answer.

    (a) $\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k^2+3}}$

    (b) $\sum_{k=1}^{\infty} \frac{5k}{(k+2)2^k}$

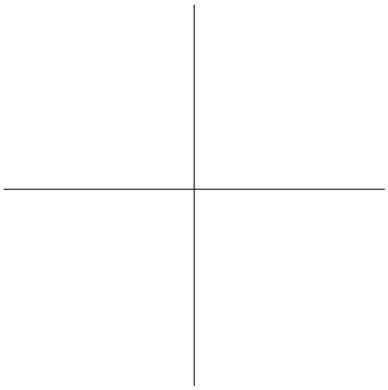
11. (8 points) Find an x-y equation for the curve given by $x = 4\cos t$, $y = \sin t$.

12. Consider the curve given by the parametric equations $x = t^2 + 3$, $y = \cos t$, where $0 \leq t \leq \pi$.

    (a) (6 points) Compute the slope of the tangent line of this curve at $t = \pi/2$.

    (b) (6 points) Set up an integral to compute the arc length of this curve. **DO NOT EVALUATE THIS INTEGRAL.**

13. (a) (6 points) Sketch the graph of $r = 1 - 2\cos(2\theta)$.



    (b) (12 points) Set up an integral to find the area of one large loop of this graph. **DO NOT EVALUATE THIS INTEGRAL.**