Name:____________________ Section:____________ Score:____________

Instructions: You must show supporting work to receive full and partial credits.

1(15pts) (a) Sketch the solid obtained by rotating the region bounded by $y = x^3, x = 1, y = 0$ about the axis $y = 1$. Use the sketch to set up a Riemann sum to approximate the volume.

(b) Use the Riemann sum to derive an integral for the volume and find the volume.

2(15pts) (a) Derive an integral for the arc length of the parabola $y = x^2$ between $x = -2$ and $x = 2$.

(b) Approximate the length by Simpson Rule SIMP(10).

(*Continue on Next Page ...*)

3(10pts) A rod of length 2 meters and density $\delta(x) = 3 - e^{-x}$ kilograms per meter is placed on the x-axis, with its ends at $x = \pm 1$. Find the coordinate of the center of mass. (Assume $\int xe^{-x}dx = -xe^{-x} - e^{-x} + C$.)

4(15pts) (a) A water tank is in the form of a right circular cylinder with height 20 ft and radius 6 ft. If the tank is half full of water, find the work required to pump all of it over the top rim. (Note that 1 cubic foot of water weighs 62.4 lb.)

(b) Find the force on the bottom of the tank.

(*Continue on Next Page ...*)

5(15pts) (a) Find the Taylor polynomials of degree $n = 2, 3$ for $f(x) = \sqrt{1+x}$ at point $x = 3$.

(b) Suppose you know that all the derivatives of some function f exists at 0, and that the Taylor series of f about $x = 0$ is

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots.$$

Find $f^{(10)}(0)$. (Note this part has no relation whatsoever with part (a).)

6(15pts) (a) Use the ratio test to find the radius of convergence of the power series

$$x + \frac{2^2}{2!}x^2 + \frac{3^2}{3!}x^3 + \frac{4^2}{4!}x^4 + \cdots.$$

(b) Find the Taylor series about $x = 0$ for $f(x) = \frac{x}{1+x^2}$ assuming the known power series for $\frac{1}{1-x}$.

(*Continue on Next Page ...*)

7(15pts) Find the exact values for the sums, finite or infinite.

(a) $\frac{3}{2} - \frac{3^2}{2^2} + \frac{3^3}{2^3} - \frac{3^4}{2^4} + \cdots + \frac{3^{11}}{2^{11}}$

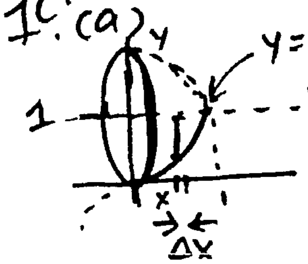
(b) $1 - \frac{3}{2} + \frac{1}{2!}\frac{3^2}{2^2} - \frac{1}{3!}\frac{3^3}{2^3} + \frac{1}{4!}\frac{3^4}{2^4} - \cdots$

2 Bonus Points: The State Flower of Nebraska is: _________

The End

Solu. Key Test 2 Math 107 Fall 02

1(15pts) (a)



a) $\Delta V = \pi 1^2 - \pi(1-y)^2 \Delta x = (\pi - \pi(1-x^3)^2)\Delta x$, $V \approx \pi - \sum \pi(1-x^3)^2 \Delta x$

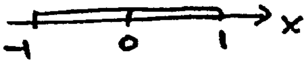
b) $V = \lim_{\Delta x \to 0} \sum (\pi - \pi(1-x^3)^2)\Delta x = \int_0^1 \pi - \pi(1-x^3)^2 dx$

$= \pi \int_0^1 1-(1-2x^3+x^6)dx = \pi(1-(x-\frac{1}{2}x^4+\frac{1}{7}x^7)\Big|_0^1)$

$= \pi(1-(1-\frac{1}{2}+\frac{1}{7})) = \pi(1-\frac{14-7+2}{14}) = \frac{5}{14}\pi$

2(15pts) (a) $\ell = \int_{-2}^{2} \sqrt{1+y'^2}\,dx = \int_{-2}^{2} \sqrt{1+4x^2}\,dx$ (b) 9.2936

3(10pts)

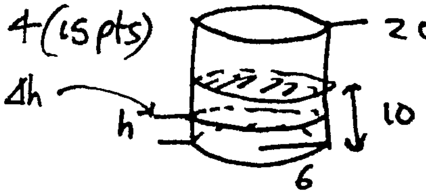


$m = \int_{-1}^{1} \delta(x)dx = \int_{-1}^{1}(3-e^{-x})dx = 3x+e^{-x}\Big|_{-1}^{1}$

$= 6+e^{-1}-e^{+1}$. $m_x = \int_{-1}^{1} x\delta(x)dx = \int_{-1}^{1}(3x-xe^{-x})dx = \frac{3}{2}x^2-xe^{-x}-e^{-x}\Big|_{-1}^{1} = -2e^{-1}$

⇒ center of mass. $\bar{x} = \frac{m_x}{m} = \frac{-2e^{-1}}{6+e^{-1}+e^{1}}$

4(15pts)



(a) Let h be the water level from bottom. Then

$\Delta W = 62.4(\pi)(6^2)\Delta h(20-h)$.

$W = \int_0^{10} 62.4\pi(36)(20-h)dh = 2246.4\pi(20h-\frac{1}{2}h^2)\Big|_0^{10}$

$= 336960\pi$ lb·ft. (b) $62.4(10)\pi(6^2) =$ ~~22464~~ $70,572.7$

5(15pts) (a) $f(x) = (1+x)^{\frac{1}{2}}$, $f(3) = 4^{\frac{1}{2}} = 2$. $f'(3) = \frac{1}{2}(1+x)^{-\frac{1}{2}} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$f''(3) = -\frac{1}{4}(1+x)^{-\frac{3}{2}} = -\frac{1}{4}\frac{1}{2^3} = -\frac{1}{32}$. $f'''(3) = \frac{3}{8}(1+x)^{-\frac{5}{2}} = \frac{3}{8}\cdot\frac{1}{2^5} = \frac{3}{256}$

$P_2(x) = 2+\frac{1}{4}(x-3) - \frac{1}{32}\cdot\frac{1}{2!}(x-3)^2$. $P_3(x) = 2+\frac{1}{4}(x-3) - \frac{1}{32}\cdot\frac{1}{2!}(x-3)^2 + \frac{3}{256}\cdot\frac{1}{3!}(x-3)^3$

(b) $f^{(10)}(0) = c_{10}\cdot(10!) = \frac{1}{10}(10!) = 9!$

6(15pts) 7 (a) $\lim_{n\to\infty} \frac{|c_n|}{|c_{n+1}|} = \lim_{n\to\infty} \frac{n^2/n!}{(n+1)^2/(n+1)!} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^2 \frac{(n+1)!}{n!} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^2(n+1) = \infty$ 2 2

$R = \infty$, it converges for all x.

8 (b) $\frac{1}{1-x} = 1+x+x^2+x^3+\cdots = \sum_{n=0}^{\infty} x^n$, $f(x) = \frac{x}{1+x^2} = x\left(\frac{1}{1-(-x^2)}\right)$

$= x[1-x^2+x^4-x^6+\cdots] = x-x^3+x^5-x^7+\cdots = \sum_{n=0}^{\infty}(-1)^n x^{2n+1}$

7(15pts) (a) $\frac{3}{2}-\frac{3^2}{2^2}+\frac{3^3}{2^3}-\frac{3^4}{2^4}+\cdots+\frac{3^{11}}{2^{11}} = \frac{3}{2}\frac{(-\frac{3}{2})^{11}-1}{-\frac{3}{2}-1} = 3((\frac{3}{2})^{11}-1)$

(b) $1-\frac{3}{2}+\frac{1}{2!}\frac{3^2}{2^2}-\frac{1}{3!}\frac{3^3}{2^3}+\frac{1}{4!}\frac{3^4}{2^4}-\cdots = e^{-\frac{3}{2}}$

~~(c) $\ln(1+x) = x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\cdots$. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots = \ln 2$~~

2 GoldenRod. give away. $a(1-x^n)$
Dt