**Name:** ─────────────────────          **TA's Name:** ─────────────

| Problem | 1 | 2 | 3 | 4 | 5 | Total |
|---------|---|---|---|---|---|-------|
| Score   |   |   |   |   |   |       |

**Instructions:** You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

**1(20pts)** Find $\frac{dy}{dx}$ for each of the functions. (**Do not simplify wherever not necessary!**)

(a) $y = \ln(e^x + \ln(e^x + 1))$

(b) $y = \dfrac{\sqrt{x}\sin x - 100}{2x^{3/2} + 5x + 1}$

**2(20pts)** (a) The graph of a function $f$ is given in Fig.2(a). In the same plate, sketch a reasonable function for its derivative $f'$.

(b) The graph of the derivative $g'$ of a function $g$ is given in Fig.2(b). In the same plate, sketch a reasonable function for $g$ itself satisfying $g(0) = 0$.
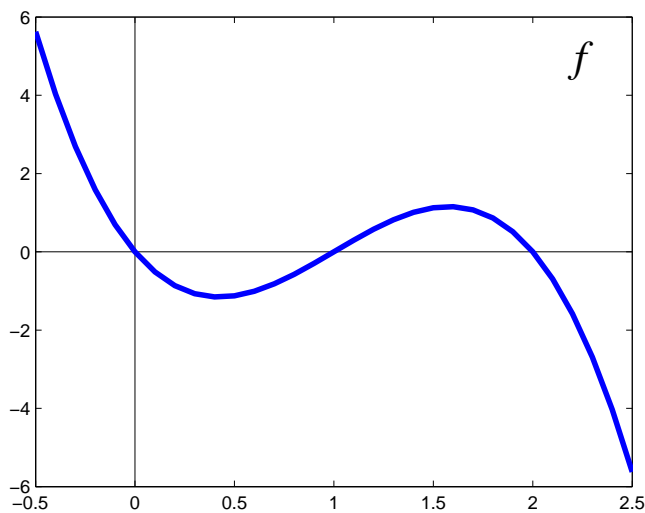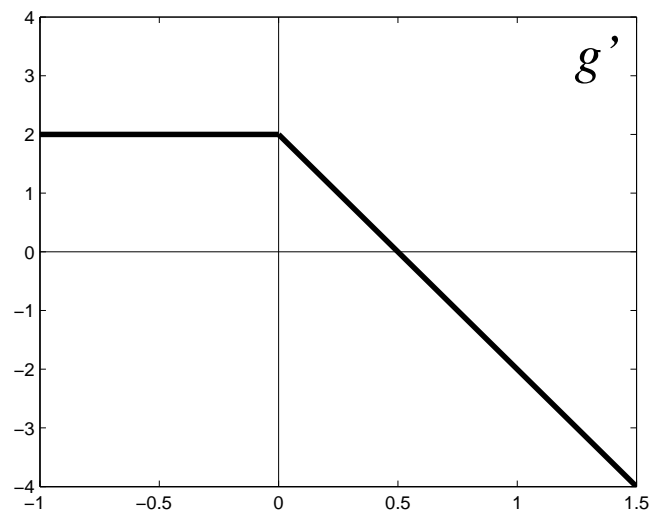


Fig.2(a)



Fig.2(b)

**3(20pts)** (a) Use implicit differentiation to find $\dfrac{dy}{dx}$ if $x$ and $y$ satisfy the equation $e^{xy} + x^2 + y^2 = 2x$.

(b) Check that the point $(1, 0)$ is on the curve defined by the same equation $e^{xy} + x^2 + y^2 = 2x$. Then find an equation of the tangent line to the curve at the point.

**4(20pts)** (a)(10pts) Use the **definition** of derivative to find the derivative $f'(2)$ analytically where $f(x) = \dfrac{2}{x+2}$. (Note: no partial points to any other methods.)

(b)(5pts) Estimate the derivative $g'(3)$, given the following information:

| $x$ | 2.8 | 2.9 | 3 | 3.1 | 3.2 |
|-----|-----|-----|---|-----|-----|
| $g(x)$ | -5 | -5.25 | -5.3 | -5.45 | -5.55 |

(c)(5pts) If the tangent line at a point of a function $y = f(x)$ is $2x - 3y = 1$ and the point is $(2, 1)$, what is $f(2)$ and $f'(2)$?

**5(20pts)** (a) Find the value of $g'(1)$ if $g(x) = x^2 f(3x)$ and $f$ satisfies $f(3) = -2$, $f'(3) = 2$.

(b) If $h(x) = x^{29} + 1000x^2$, what is its 99th derivative $h^{(99)}$?

(d) The graph of the second derivative $f''$ of a function is given in the figure, sketch a reasonable graph for the first derivative $f'$, and a reasonable graph for the function $f$ itself. Superimpose both graphs in the same figure, and label them with $f'$, $f$ respectively.
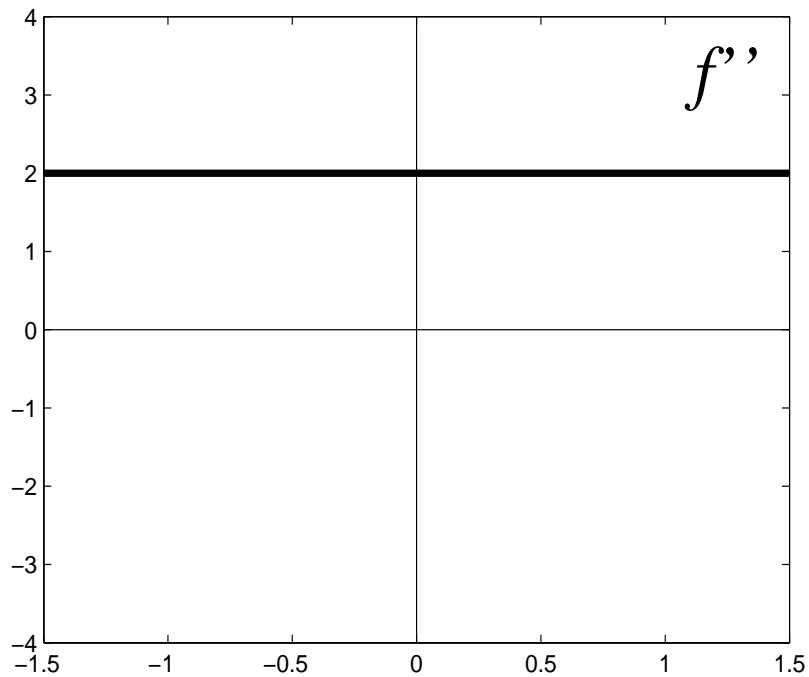


**2 Bonus Points**: The state bird of Nebraska is: _____    (... *The End*)