

Name: _____

TA's Name: _____

Instructions: You must show supporting work as much as possible to receive full and partial credits.
No text book, notes, formula sheets allowed.

1(18pts) Let $y = f(x) = x^5 + 37x$.

(a) Find an equation for the tangent line of the function $y = f(x)$ at $x = 1$.

(b) Find the value of dy when $x = 1$ and $\Delta x = 0.2$.

(c) It is given that for a function $y = g(x)$, $g(2) = 1$, $g'(2) = 0.2$. Approximate the value $g(1.8)$ and justify your answer.

2(16pts) Sand falls from a conveyor belt as the rate of $10 \text{ m}^3/\text{min}$ onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast is the radius changing when the pile is 4 m high?

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3(16pts) Find the absolute extrema of the function $y = f(x) = x^{4/5}(5 - x)$ in the interval $[-1, 1]$. (Calculator read-off answers receive no credit.)

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4(16pts) Find the following limits. (Calculator read-off answers receive no credit.)

(a) $\lim_{x \rightarrow 0^+} \frac{\ln(x^2 + 2x)}{\ln x}$

(a) $\lim_{x \rightarrow +\infty} x^2 e^{-x}$

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5(18pts) The first and second derivatives of the function $y = xe^{1/x}$ are given as

$$y' = e^{1/x} \left(1 - \frac{1}{x} \right), \quad y'' = \frac{e^{1/x}}{x^3}$$

Sketch the graph of the function, including these information: intercept points, horizontal and vertical asymptotes, extrema and intervals of monotonicity, inflection points and intervals of concavity. (Calculator read-off answers receive no credit.)

6(16pts) A rectangle has its base on the x -axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?