

## Spring 2011 Math106 Project: Approximating $\sqrt{2}$

1. It is generally accepted that the Pythagoreans in the late 5th BC were the first to discover irrational numbers such as  $\sqrt{2}$  which is the hypotenuse of a right triangle of equal sides of unit length and which is not a integer or ratio of two integers. However, a Babylonian stone tablet dating from 1900 BC shows two approximating values: 1.4, 1.414285 of  $\sqrt{2}$ . The first part of the project is to derive an approximating formula that the Babylonian might have used to derive their result. Here is an outline of the steps to derive the formula.

Let  $x = \sqrt{a}$  be the exact value of  $\sqrt{a}$  for a given  $a$  ( $a = 2$  for the specific case of  $\sqrt{2}$ ) and let  $x_0 \sim x$  be an initial approximation ( $x_0 = 1.4$  for the special case  $\sqrt{x}$ ). Let  $x_1 = x_0 + \delta$  be the next approximating value for a presumably small value  $\delta$  so that  $|x^2 - x_1^2| = \delta^2$  holds. Determine  $\delta$  and hence the formula for  $x_1$ . Use the formula to find the first few approximations  $x_i$ .

2. Section 4.7 is not covered by the syllabus. You need to teach yourself on Newton's Method (1671) from that section. Summarize the method and then show that the iterative formula by Newton's Method for approximating  $\sqrt{a}$  is exactly the same you derived in part 1 above.

3. To improve the formula  $y_1 = f(x_0)$  from part 1 (part 2 also), consider a revised approximation  $x_1 = f(x_0) + \sigma$  so that  $|x^2 - x_1^2| = \sigma^2$  holds. Derive  $\sigma$  and hence the revised formula  $x_1 = g(x_0)$  (due to Alkalsâlî around 1450 and Briggs 1624). Use the formula to find the first few approximations and compare the method and result to that of part 1.

Your report must be self-contained, having a title followed by the authors, an introduction paragraph on whatever you see fit to include, a body of method, analysis, and result, and a concluding paragraph. It must not be fragmented into 3 parts as the project statement above, but in a seamless flow of narrative, graph, formula, and table of numbers. In fact, you should organize your report in any order you prefer, not necessarily in the same order as the project statement above. It must be produced by a computer word processor including the mathematical formulas, and be turned in a hard copy. You must avoid using phrases such as "we are asked to do such and such" or "the project asks us this and that". If your report reads like a chapter section from the textbook, you probably have done it 90% right. In other words, you must assume the reader of your report to be a student of this class. Above all, your project must demonstrate an effort in good-faith.

Your project will be graded by these categories: format, writing(explanations), merit, and effort for 5 points each. No shared materials are allowed. Zero point will be given to any project that violates this no-copying rule.