Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) (a) For the definite integral $\int_1^2 \ln x \, dx$, find (not by your calculator program) the left point Riemann sum L_4.

(b) Find the exact value of the limit

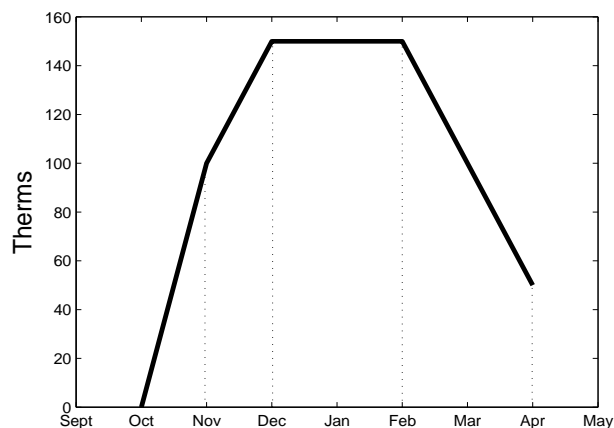$$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{3k}{n} + 1 \right) \frac{2}{n}$$

(Usable identities: $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$, $1^2 + 2^2 + \cdots + n^2 = \frac{(2n+1)n(n+1)}{6}$)

2(10pts) (a) If $\int_0^4 f(x)dx = 4$, what is the average value, f_{ave}, of f over the interval $[0,4]$?

(b) If $\int_0^4 f(x)dx = 4$ and $\int_2^4 f(x)dx = 2$, what is $\int_0^2 f(x)dx$?

3(10pts) The graph below shows the natural gas usage (in therms) for heating my house from October to March. Find my average monthly usage over these 6 months.

4(20pts) Some values of a function $f(x)$ is given below:

x	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
$f(x)$	0.15	0.1	0.0	-0.1	-.2	-0.32	-0.58	-0.58	-0.6

Approximate the value of the integral $\int_{-1}^{1} f(x)dx$ by the following Riemann sums:

(a) The left point sum L_4.

(b) The right point sum R_4.

(c) The midpoint sum M_4.

(d) The trapezoid sum T_4.

(e) The Simpson sum S_4.

5(25pts) Evaluate the following: (No calculator is allowed for these problems.)

(a) $\displaystyle\int \sec^2 2x + 3\sin 3x \ dx$

(b) $\displaystyle\int_0^{\pi/4} \tan x \ dx$

(c) $\displaystyle\int \frac{x^2 + x + 1}{\sqrt{x}} \ dx$

(d) $\displaystyle\int \frac{x+1}{x^2 + 2x} \ dx$

6(10pts) Let $f(x) = \dfrac{\sin(\pi x^2)}{x^2+1}$.

(a) What is $\displaystyle\int_0^{1/2} f'(x)\,dx$?

(b) What is $\dfrac{d}{dx}\displaystyle\int_0^{x^2} f(t)\,dt$?

7(10pts) A corner of a large room is to be fenced off temporarily for some circus animals. The fencing gate is 10 ft long, and it is put against the wall to enclose a right triangle area. Find the dimensions of the triangle so that its area is maximal.

10' fence

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2 Bonus Points: The state fish of Hawaii is _____.   (... *The End*)