Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) (8 points each) Find $\frac{dy}{dx}$ for each of the functions. (**Do not simplify wherever not necessary!**)

(a) $y = \arctan \dfrac{\sqrt{x^2 + 1}}{x}$

(b) $y(1 + xy^3) = x^4 + 1$

2(15pts) (a) Find the exact values of all critical points of the function $f(x) = \dfrac{\ln x}{x}$.

(b) Use the second derivative test to determine the critical points as local maxima or local minima. (Read-out answers from calculators receive no points.)

3(15pts) Find the limits. If L'Hopital Rule is used, you must verify the undetermined type of the limit before the rule is applied. (Calculator answers are not accepted.)
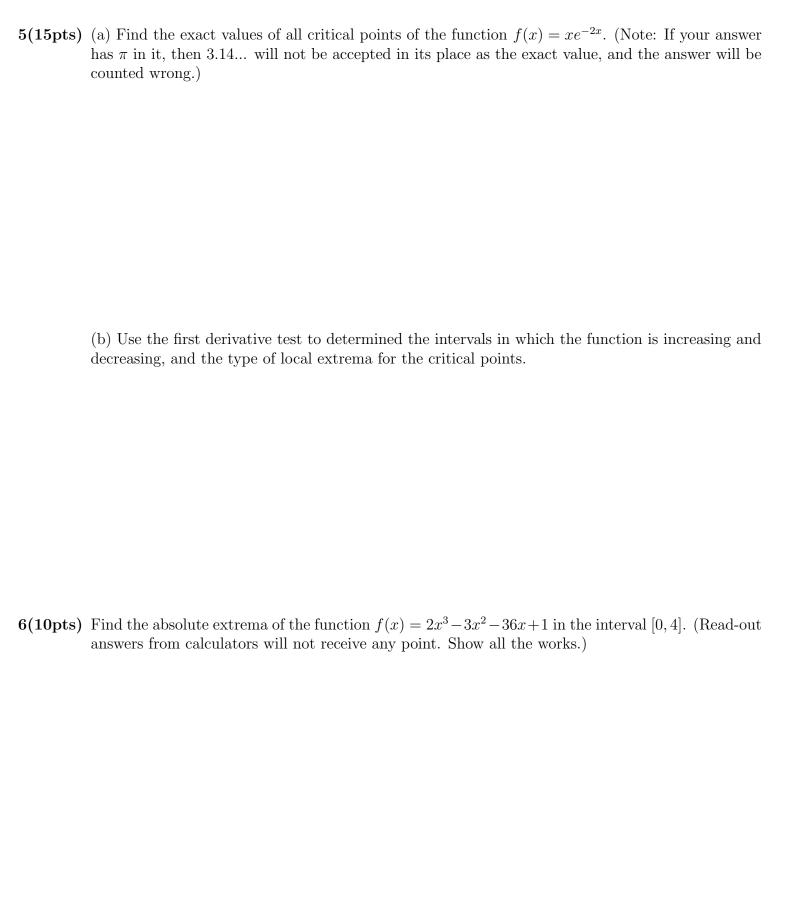
(a) $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3}$

(b) $\lim\limits_{x \to \infty} \dfrac{e^{\sqrt{x}}}{x}$

4(15pts) (a) Find an equation of the tangent line to the function $y = f(x)$ at $x = 1$ if it satisfies these conditions: if $f(x) = e^{3g(x)}$ with $g(1) = 0, g'(1) = -1$.

(b) Use the linear approximation to estimate the value $g(1.98)$ given the following information:

x	1.8	1.9	2	2.1
$g(x)$	-.3	-3.3	-3.8	-4

5(15pts) (a) Find the exact values of all critical points of the function $f(x) = xe^{-2x}$. (Note: If your answer has π in it, then 3.14... will not be accepted in its place as the exact value, and the answer will be counted wrong.)

(b) Use the first derivative test to determined the intervals in which the function is increasing and decreasing, and the type of local extrema for the critical points.

6(10pts) Find the absolute extrema of the function $f(x) = 2x^3 - 3x^2 - 36x + 1$ in the interval $[0, 4]$. (Read-out answers from calculators will not receive any point. Show all the works.)

7(15pts) Sketch a graph of the function

$$f(x) = \frac{2x}{1-x},$$

including all significant features of the functions: domain of the function, x-intercept, y-intercept, vertical and horizontal asymptotes, local extrema, interval of increasing and decreasing, inflection points, interval of concavities. A calculator assisted graph without accompanying analysis will not received any credit.