Math 106-Sec 25X Spring '04 **Exam 3** **Score:** _____

Name: _____ **TA's Name:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(14pts) Sketch a graph of a function that satisfies the following conditions:

(i) $\lim_{x\to -1^-} f(x) = \infty,\ \lim_{x\to -1^+} f(x) = \infty,\ \lim_{x\to \infty} f(x) = 1.$

(ii) $f(0) = 0.$

(iii) $f'(0) = f'(2) = 0,\ f'(x) > 0$ on $(-\infty, -1),\ (0, 2)$ and $f'(x) < 0$ on $(-1, 0), (2, \infty).$

(iv) $f''(x) > 0$ on $(-\infty, -1),\ (-1, 1),\ (2.5, \infty)$ and $f''(x) < 0$ on $(1, 2.5).$

2(6pts) Use the definition to find $\int_0^1 x dx$. (*Suggestion*: Evaluate the limit of the right end point Riemann sum as the number of partition n tends to infinity.)

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3(12pts) (6 points each) (a) Determine if this function $f(x) = \dfrac{e^x - e}{x^2 - 1}$ has a vertical asymptote. If so what is it? If not why not? (Do not sketch the curve and an answer based on calculator work does not receive any credit.)

(b) Determine if the function $g(x) = \dfrac{2x^2 + 1}{x^2 + 2x + 2}$ has a horizontal asymptote. If so find them all.

4(8pts) (4 points each) Suppose that $G(t)$ represents the gasoline usage in gallons of your car per minute going from Lincoln to Omaha that takes about 50 minutes. Describe what each of the following quantities represents and in what unit each quantity is.

(a) $\displaystyle\int_0^{50} G(t)dt$

(b) $\dfrac{1}{50}\displaystyle\int_0^{50} G(t)dt$

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5(20pts) (5 points each) Find the antiderivatives of the following functions:

(a) $f(x) = 4x^{10.8} + \dfrac{2}{\sqrt{x}}$

(b) $g(x) = \dfrac{x^2 + x \sin x}{x}$

(c) $h(x) = (2x + 1)^2$

(d) If $F(x) = \dfrac{\sin x^2}{x^2}$, what is $\displaystyle\int F'(x)dx$?

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6(6pts) (3 points each) (a) If $F(x) = \int \tan e^x dx$, what is $\dfrac{dF(x)}{dx}$?

(b) If $\int_1^2 f(x)dx = 2$ and $\int_1^3 f(x)dx = 3$, what is $\int_2^3 f(x)dx$?

7(14pts) (3 points each except for the last) Some values of a function $f(x)$ is given below:

x	0.5	0.75	1	1.25	1.5	1.75	2.0	2.25	2.5
$f(x)$	0.4	0.1	0	-0.12	-0.38	-0.38	-0.4	-0.4	-0.4

Approximate the value of the integral $\int_{0.5}^{2.5} f(x)dx$ by the following Riemann sums:

(a) The left point sum L_4.

(b) The right point sum R_4.

(c) The midpoint sum M_4.

(d) The trapezoid sum T_4.

(e) The Simpson sum S_4.

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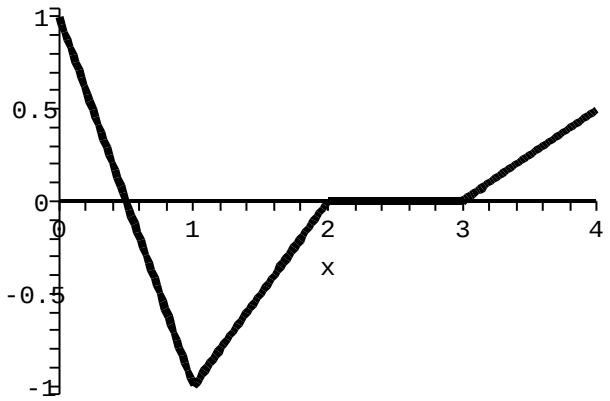
8(14pts) A closed cylindrical can with a circular base is to be constructed so that its volume is 8π cubic feet. The material for the sides cost \$2 per square foot while the material for the top and bottom cost \$1 per square foot. Find the dimensions of the can that minimize the cost.

9(6pts) (3 points each) A function $f(x)$ on $[0, 4]$ is shown below.



(a) Fill in the blank $\int_0^2 f(x)dx =$ ____.

(b) Fill in the blank $\int_2^3 f(x)dx =$ ____.

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2 Bonus Points: The letter grade equivalence to a numerical test score of 87 is an (a) A, (b) A-, (c) B+, (d) none of the above. (*... The End*)