Math 106-Sec 25X Spring '04 **Exam 2** **Score:** ____________

Name: ______________________________ **TA's Name:** ________________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(8pts) Find the equation of the tangent line to the graph of the curve defined by the equation $\ln y + xy - x^3 + 6 = 0$ at the point $(2, 1)$.

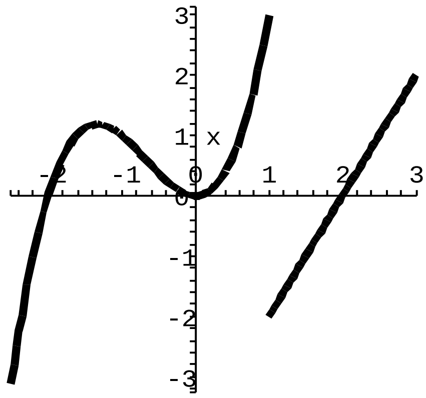
2(12pts) (6 points each) Find the limits (If you use L'Hospital's rule make sure to justify its use):

(a) $\displaystyle\lim_{x\to 2} \frac{\cos(x^2-4)-1}{x-2}$

(a) $\displaystyle\lim_{x\to 0} x \cot x$

(*Continue on Next Page ...*)

3(20pts) (7pts each for (a) and (b)) The derivative $f'(x)$ of a function $f(x)$ on an interval is shown below.



(a) **Find** all the critical points in the interval and **classify** the critical points by the First Derivative Test.

(b) Determine the intervals on which $f(x)$ is concave up and concave down, and find all inflection points in the interval.

(c) In the space next to the given graph, sketch a plausible graph of $y = f(x)$ featuring all important elements of the function.

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4(8pts) Some values of a function $f(x)$ is given below:

x	0.5	0.75	1	1.25	1.5
$f(x)$	0.4	0.1	0	-0.12	-0.38

Approximate first the derivative $f'(1)$ and then use linear approximation to approximate the value of $f(1.1)$.

5(12pts) Find the absolute maximum and minimum values of $f(x) = 3x^4 + 4x^3 - 36x^2 + 3$ on the interval $[0, 3]$.

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6(6pts) If $f(2) = 1, f'(2) = 5, g(1) = 2, g'(1) = -3$ and $h(x) = \ln[f(g(x))]$, what is $h(1)$ and $h'(1)$?

7(14pts) (7 points each) Find $\frac{dy}{dx}$ for each of the functions. (**Do not simplify!**)

(a) $y = \dfrac{1 + \cos x^2}{3^x \ln x}$

(a) $y = \tan^{101}(\sqrt{x} + \sin e^{x^2})$

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8(10pts) Let $f(x) = 2x^3 + 3x^2 - 12x + 1$ and it is given that $x = -2, 1$ are the critical points. Use the 2nd Derivative Test to determine the local extrema.

9(10pts) Find the value of c satisfying the conclusion of the Mean Value Theorem for the function $f(x) = x^3 - x^2 - x + 1$ on the interval $[0, 2]$.

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2 Bonus Points: Who has won the most delegates so far for Democratic Party's presidential nominee this year? (a) Howard Dean, (b) John Kerry, (c) John Edwards. (... *The End*)

Math106 Exam 2 Soln Key, Spring '04

1 (8pts) $\ln y + xy - x^3 + 6 = 0$. At $(2,1)$, $\ln 1 + 2(1) - 2^3 + 6 = 0$. the point is on the curve. Implicit Differentiation: $\frac{1}{y}\frac{dy}{dx} + y + x\frac{dy}{dx} - 3x^2 = 0$

$\frac{1+xy}{y}\frac{dy}{dx} = 3x^2 - y$, $\frac{dy}{dx} = \frac{(3x^2-y)y}{1+xy} \underset{@(2,1)}{=} \frac{11}{2}$. Tangent line: $y = 1 + \frac{11}{2}(x-2)$, $y = 5.5x - 10$

2 (12pts) (6 each) (a) $\lim_{x\to 2} \frac{\cos(x^2-4)-1}{x-2} \underset{\frac{0}{0}\text{ type}}{=} \lim_{x\to 2} \frac{-\sin(x^2-4)(2x)}{1} = 0$

(b) $\lim_{x\to 0} x\cot x = \lim_{x\to 0} \frac{x\cos x}{\sin x} \underset{\frac{0}{0}\text{ type}}{=} \lim_{x\to 0} \frac{\cos x - x\sin x}{\cos x} = \frac{1-0}{1} = 1$ ($0\cdot\infty$)

3 (20pts). (a) c.pts: $x = -2, 0, 1, 2$

f': − (−2) + (0) + (1) − (2) +

f: ↘ ↗ ↗ ↘ ↗ ; loc min (−2), none (0), loc max (1), loc min (2)

(b) f': ↗ ↘ ↗ ↗ ; f'': + (−2) + (−1) − (0) + (1) + (2)

f: ∪ inf ∩ inf ∪ none ∪

(c)

4 (8pts) $f'(1) \doteq \frac{f(1.25)-f(1)}{1.25-1} = \frac{-0.12-0}{.25} = -0.48$. $f(1.1) \approx f(1) + f'(1)(1.1-1) = 0 - 0.48(0.1) = -0.048$

5 (12pts) $f(x) = 3x^4 + 4x^3 - 36x^2 + 3$, $f' = 12x^3 + 12x^2 - 72x = 12x(x^2+x-6) = 0$

$\Rightarrow x = 0$, $x^2 + x - 6 = 0 \Rightarrow (x+3)(x-2) \Rightarrow x = -3, 2$.

x	0	2	3
f	3	−61 (abs min)	30 (abs max)

6 (6pts). $f(2)=1$, $f'(2)=5$, $g(1)=2$, $g'(1)=-3$,

$h(x) = \ln[f(g(x))]$, $h(1) = \ln[f(g(1))] = \ln[f(2)] = \ln(1) = 0$

$h'(1) = \frac{1}{f(g(x))} f'(g(x))\, g'(x)\big|_{x=1} = \frac{1}{1}(5)(-3) = -15$

7 (14pts). (a) $y = \frac{1+\cos x^2}{3^x \ln x}$, $y' = \frac{(-\sin x^2(2x))3^x \ln x - (1+\cos x^2)[3^x \ln 3 \ln x + 3^x/x]}{3^{2x}(\ln x)^2}$

(b) $y = \tan^{101}(\sqrt{x} + \sin e^{x^2})$, $y' = 101\tan^{100}(\sqrt{x} + \sin e^{x^2})(\frac{1}{2\sqrt{x}} + \cos e^{x^2} \cdot e^{x^2} \cdot 2x)$

8 (10pts) $f(x) = 2x^3 + 3x^2 - 12x + 1$, c pt. $x = -2, 1$,

$f'(x) = 6x^2 + 6x - 12$, $f'' = 12x + 6$

c.pt	−2	1
f''	−18	18 > 0
f	loc max	loc min

9 (10pts) $f(x) = x^3 - x^2 - x + 1$ on interval $[0,2]$

$\frac{f(2)-f(0)}{2-1} = \frac{8-4-2+1}{1} = 3 = f'(x) = 3x^2 - 2x - 1$

$\Rightarrow 3x^2 - 2x - 4 = 0$, $x = \frac{2 \pm \sqrt{2^2 - 4(3)(-4)}}{2(3)} = \frac{2 \pm \sqrt{64}}{6} = \frac{2 \pm 8}{6} = \begin{cases} 5/3 \\ -1 \end{cases}$

$\Rightarrow c = 5/3$

2 Bonus (b)