1(20pts) (a) A function \( f \) is given at the those \( x \) values shown in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
<th>1.1</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1.28</td>
<td>1.62</td>
<td>2</td>
<td>2.42</td>
<td>2.88</td>
</tr>
</tbody>
</table>

i. Use difference quotients to approximate the 1st derivative \( f'(1) \).

ii. Use the tangent line approximation to estimate \( f(0.95) \).

(b) On the graph \( f \)

i. Label the points where the derivatives are zero as \( A_1, A_2, \ldots \)
ii. Label the points where the derivatives do not exist as \( B_1, B_2, \ldots \)
iii. Label the points where the 2nd derivatives are zero as \( C_1, C_2, \ldots \)
iv. List all the intervals on which \( f \) is increasing.
v. List all the intervals on which \( f \) is concave up.

(c) Use the definition of derivative to show that \( \frac{d}{dx} \left( \frac{1}{x} \right) = -\frac{1}{x^2} \).
2(25 pts) (a) How many partitioning subintervals are needed over interval \([0, 2]\) in order to have an approximating Riemann sum within 0.01 of the exact definite integral \( \int_0^2 \frac{1}{\sqrt{8 - x^2}} \, dx \)?

(b) Approximate the integral \( \int_0^2 \frac{1}{\sqrt{8 - x^2}} \, dx \) by the right Riemann sum with the same partition number you found in (a).

(c) Use your answer in part (b) to estimate the average value of \( f(x) = \frac{1}{\sqrt{8 - x^2}} \) in the interval \([0, 2]\).

(d) Does the Trapezoidal Sum over estimate or under estimate the exact definite integral? Explain your answer with a graph.
3(30pts) Find the derivative of each function.

(a) $3e^{5x} + \sin x$

(b) $\frac{2}{x^2} + \sqrt{x^2 + 1}$

(c) $\frac{\sin(x + 100)}{x^3 + x + 1}$

(d) Find the second derivative $f''(x)$ of $f(x) = \cos(e^x)$.

(e) Find the exact value of $f'(\pi)$, if $f(x) = x^2 \sin x$. 
4(25pts) (a) Use the left sum to estimate the definite integral $\int_{1}^{2} f(x)dx$ for the function given in the table

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>1.28</td>
<td>1.62</td>
<td>2</td>
<td>2.42</td>
<td>2.88</td>
<td>3.40</td>
</tr>
</tbody>
</table>

(b) The derivative $F'$ of a function is given in the graph. Assume $F(0) = 1$, use the Fundamental Theorem of Calculus to find these values of $F$:

(i) $F(0.5)$

(ii) $F(1)$

(iii) $F(2.5)$

(iv) $F(4)$

(c) Find the exact value of $\int_{0}^{e} (2x - 1)dx$, using the Fundamental Theorem of Calculus.

3 Bonus Points: True or false?

(a) You cannot integrate a function if it has a discontinuous jump even though it is continuous everywhere else.

(b) You can differentiate a function so long as it is continuous.

The End
1. (a) \( f'(1) = \frac{f(1.1) - f(1)}{0.1} = \frac{2.42 - 2}{0.1} = 4.2 \) (ii) \( f(0.95) = f(1) + f'(1)(0.95 - 1) = 2 + 4.2(-0.05) = 1.79 \) (approx. 1.81)

(b) \[ \frac{d}{dx} \left( \frac{1}{x} \right) = \lim_{h \to 0} \frac{1 + x - x}{h} = \lim_{h \to 0} \frac{x - (x + h)}{h} = \lim_{h \to 0} \frac{-h}{h(x + h)} = \lim_{h \to 0} \frac{-1}{x(x + h)}(x) \]

2. (a) \( f(x) = \frac{1}{18 - x^2} \) ∃ \( \int_{0}^{2} f(x) \, dx \), Using sum of n-partition, required

(b) \( n = \frac{1}{f(2) - f(0) \cdot 2} = 29.29, \Rightarrow n = 30 \)

(c) \( \int_{0}^{2} \frac{1}{18 - x^2} \, dx = 0.79 \approx 0.795 \)

(d) \( \frac{1}{2} \int_{0}^{2} \frac{1}{18 - x^2} \, dx = \frac{0.79}{2} = 0.395 \)

3. (a) \( (3e^{5x + \sin x})' = 3e^{5x + \sin x} \cdot (5 + \cos x) \)

(b) \( \frac{2}{x^2 + \sqrt{x + 1}}' = (2x^{-2} + (x^{-1})^{1/2})' = -2x^{-3} + \frac{1}{2}(x^{-1})^{3/2} = -\frac{3}{x^3} + \frac{x}{x^{3/2}} \)

(c) \( (\sin(x + \cos x))' = \cos(x + \cos x) \cdot \frac{x + 1}{x^2 + x + 1} \)

(d) \( f(x) = (\cos(e^x))' = -\sin(e^x) \cdot e^x, f'' = -[\cos(e^x) \cdot e^2 + \sin(e^x) \cdot e^x] \)

(e) \( f(x) = x^2 \sin x, f'(x) = 2x \sin x + x^2 \cos x, f'(1) = 2 \pi \sin \pi + \pi^2 \cos \pi = -\pi^2 \)

4. (a) \( \int_{1}^{2} f(x) \, dx = (1.28 + 1.62 + 2 + 2.82 + 2.82)(6,2) = 10.04 \)

(b) (i) \( F(0.5) = F(0) + \int_{0}^{0.5} f(x) \, dx = 1 + \frac{1}{2}(-1)(1) = \frac{1}{2} \) (ii) \( F(1) = 1 \)

(iii) \( F(2.5) = 1 + \frac{1}{2}(-1)(1) = \frac{1}{2} \) (iv) \( F(4) = 2.5 + \frac{1}{2} \cdot \frac{1}{2} = \frac{17}{8} \)

(c) \( \int_{0}^{e} x \, dx = \frac{e^2}{2} \int_{0}^{e} e^x - e = e^e - e = e(e-1) \)

Bonus. (a) False (b) False