Math 106 Sec.351–353 **Hour Exam 1** **Feb. 14, 2002**

Name:_____ **Score:**_____

Instructions: You must show supporting work to receive full and partial credits.

1(20pts) (a) Use your calculator to graph the function $f(x) = \dfrac{2x^2 - x}{x^2 + 3x + 2}$ and copy the graph below.

(b) Find the x-intercept and y-intercept of the function.

(c) Find the domain of the function and state where the function is not defined.

(d) Find the vertical asymptotes and show your work aside from your calculator graph.

(e) Find the horizontal asymptote and show your work aside from your calculator graph.

2(20pts) (a) Simplify the expression as much as possible

$$\frac{\log A^2 - 2\log B}{\log A^2 + \log \frac{1}{AB}}.$$

(b) Solve for x from $7^{x+2} = e^{17x}$.

(c) Circle the dominating function as $x \to +\infty$ in each pair.

(i) $1.1^{0.1x}$, $x^{1.1}+1,000$ (ii) $\sqrt{x}+1,000\ln x$, $\log x$ (iii) $\log x$, $\ln x$

(d) The graph of f and g are given, estimate $f(g(2))$.

3(20pts) (a) What is the domain of $y = f(x) = \dfrac{2}{\sqrt{x}+1}$. What is the range?

(b) Explain why $y = f(x) = \frac{2}{\sqrt{x}+1}$ is invertible.

(c) Find the inverse function $y = f^{-1}(x)$.

(d) Use the expression found in (c) to evaluate $f^{-1}(\frac{2}{3})$.

4(20pts) Given the graph of $y = f(x)$,

(a) Sketch $1.5f(x)$.

(b) Sketch $f(1.5x)$.

(c) Sketch $f^{-1}(x)$.

(d) Sketch $\frac{1}{f(x)}$.

5(20pts) (a) Find the equation for the line through $(2, 3)$ and perpendicular to $2x + 3y - 1 = 0$.

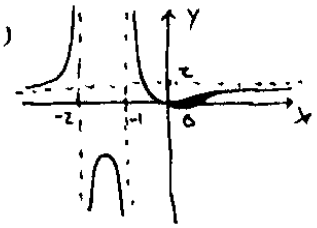
(b) Find the formula of a quadratic function $ax^2 + bx + c$ that fits the graph

(c) Find possible formulas for the graphs of (i,ii,iii).

Bonus 2 pts for taking Test 1 on Valentine's Day.

Math 106. Sec. 351-353. Test 1. Solu. Key

1 20pts (4pts each)

(a) 

(b) x-intercept: $f(x)=0 \Rightarrow 2x^2-x=x(2x-1)=0$
$x=0,\ x=\frac{1}{2} \Rightarrow \boxed{(0,0),\ (\frac{1}{2},0)}$
Y-intercept: $f(0)=0. \Rightarrow \boxed{(0,0)}$

(c) $x^2+3x+2=(x+1)(x+2)=0,\ x=-1,-2.$
Domain = all x except -1, -2, at which f is not defined.

(d) vertical asymptotes: $x^2+3x+2=0 \Rightarrow \boxed{x=-1},\ \boxed{x=-2}$

(e) horizontal asymptotes: $\lim_{x\to\pm\infty} f(x) = \lim_{x\to+\infty} \frac{2x^2}{x^2} = 2. \Rightarrow \boxed{y=2}$

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2. 20pts

(a) (6) $\dfrac{\log A^2 - 2\log B}{\log A^2 + \log \frac{1}{AB}} = \dfrac{\log \frac{A^2}{B^2}}{\log \frac{A^2}{AB}} = \dfrac{2\log \frac{A}{B}}{\log \frac{A}{B}} = 2$

(b) (6) $7^{x+2} = e^{17x}.\ (x+2)\ln 7 = 17x \Rightarrow x = \boxed{2\ln 7/(17-\ln 7)}$

(c) (6) (i) $\boxed{1.1^{0.1x}}$ (ii) $\boxed{\sqrt{x}+1,000\ln x}$ (iii) $\boxed{\ln x}$

(d) (3) $g(2)\approx 1,\ f(1)\approx 2.25. \Rightarrow f(g(2))\approx \boxed{2.25}$

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3 20pts (5pts each)

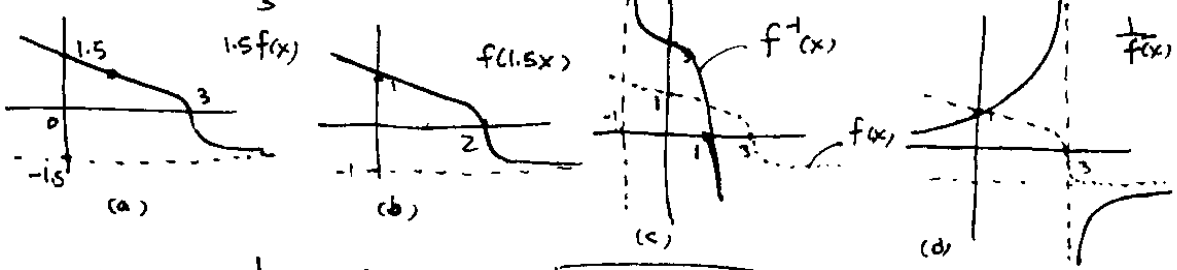
(a) $f=\frac{2}{\sqrt{x}+1}$, Domain $\boxed{x\geq 0}$, Range $\boxed{(0,2]}$ $(0<y\leq 2)$

(b) $\sqrt{x}\nearrow \Rightarrow \sqrt{x}+1\nearrow \Rightarrow \frac{2}{\sqrt{x}+1}\searrow$, monotone decreasing $\Rightarrow$ f invertible

(c) $y=\frac{2}{\sqrt{x}+1} \Rightarrow \frac{2}{y}=\sqrt{x}+1 \Rightarrow \frac{2}{y}-1=\sqrt{x} \Rightarrow x=(\frac{2}{y}-1)^2 \Rightarrow \boxed{f^{-1}(x)=(\frac{2}{x}-1)^2}$

(d) $f^{-1}(\frac{2}{3})=(\frac{2}{\frac{2}{3}}-1)^2=\boxed{1}$

4 20pts (5pts each)

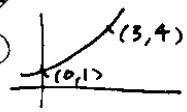


5 20pts

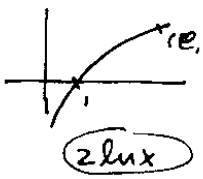
(a) (6) slope $=-\frac{1}{2/3}=-3/2$. line: $\boxed{y-3=\frac{3}{2}(x-2)}$

(b) (6) $f(x)=-x(x+5)=-x^2-5x.$

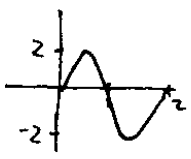
(c) (6) (3 each)



$4^{x/3}\ (=e^{\frac{\ln 4}{3}x})$



$\boxed{2\ln x}$



$2\sin(2\pi\frac{x}{2})$
$=\boxed{2\sin(\pi x)}$

End.