Math 106-250 Spring '15 **Exam 3** **Score:** _____

Print Your Name: _____ **TA's Name:** _____

Instructions: You must show supporting work as much as possible to receive full and partial credits. No text book, notes, formula sheets allowed.

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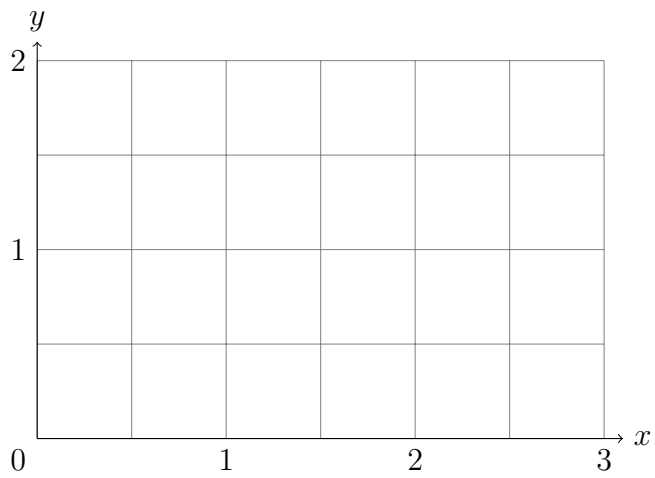
1. (**10 pts**) Consider the definite integral $\int_1^2 \ln(x+1)dx$.

   (a) Estimate the value using the right-hand sum with $n = 2$, i.e. R_2.

   (b) Sketch the function $f(x) = \ln(x+1)$ and a diagram that represents R_2. Is your estimate an overestimate or underestimate?



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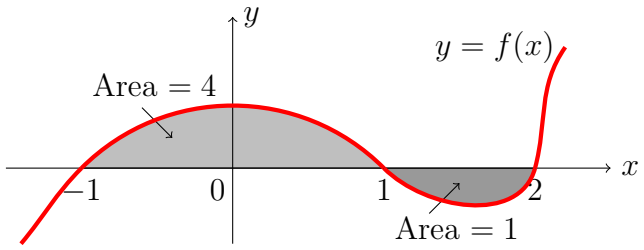
2. (**10 pts**) (a) Find $\int_{-1}^2 f(x)dx$ for the function f shown.



   (b) Find the solution to the differential equation $\frac{dy}{dx} = 2x + 1$ that also satisfies $y(0) = 1$.

(*Page 1. Continue on Next Page ...*)

3. (**20 pts**) Find the exact value or the indefinite integral of the following.

(a) $\displaystyle\int_0^1 \frac{2x-1}{x^2-x+1}dx$

(b) $\displaystyle\int \sqrt{1+\sqrt{x}}dx$

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4. (**10 pts**) An object's motion on the plane is given by this parameterized curve

$$x(t) = t - \sin(t), \quad y(t) = 1 - \cos(t)$$

in meter and second.

(a) Find the speed of the object at time $t = \pi$ second.

(b) Find an equation for the tangent line to the curve at $t = \pi$.

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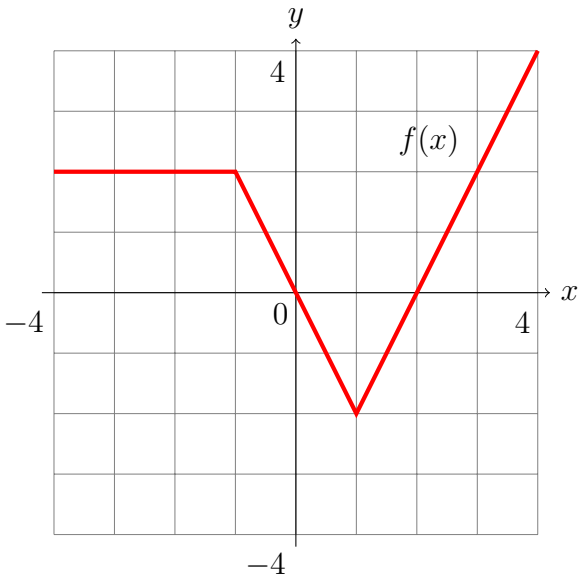
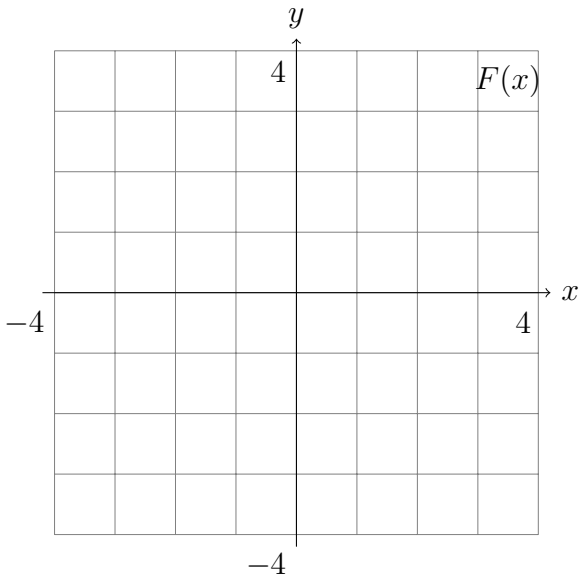
5. (**15 pts**) The graph of a function f is given below.





(a) (5 pts) Sketch the function F which satisfies $F'(x) = f(x)$ and $F(0) = 1$.

(b) (5 pts) Find the exact value of $F(2)$.

(c) (5 pts) Find also the exact values of $F(-1),\ F(4)$.

(*Page 3. Continue on Next Page ...*)

6. (**20 pts**) Find the limit, using L'Hopital's Rule if applicable.

(a) $\lim_{x\to\infty} \dfrac{\ln(e^x+10)}{2x+5}$

(b) $\lim_{x\to 0} \dfrac{e^x-1-x}{x^2}$

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7. (**15 pts**) Let $F(x) = \int_1^{x^2} \sin(t^2)dt$.

(a) (10 pts) Find $F'(x)$.

(b) (5 pts) Find all critical points of $F(x)$.

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2 point Bonus Question: The name of Nebraska State River is _____