Name: _____   TA's Name: _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(18pts) (9 points each) Find $\frac{dy}{dx}$ for each of the functions. (**Do not simplify wherever not necessary!**)

(a) $y = [1 + \ln(1 + \arcsin(x^2))]^3$

(b) $xy^2 + 1 = \tan^{-1} y$

2(16pts) (a) A function $y = f(x)$ is defined for all x. Find its critical points if its derivative is given as $f'(x) = \dfrac{x+2}{(x-1)^{1/3}}$. Use the first derivative test to classify the critical points as local maxima and local minima.

3(16pts) A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

4(16pts) (8 points each) (a) Find the linearization $y = L(x)$ of $f(x) = \tan x$ at $x = \pi/4$.

(b) Find the average rate of change for the function $f(x) = x^2$ in the interval $[0, 1]$, and then find a point $x = c$ in $(0, 1)$ so that the instantaneous rate of change $f'(c)$ is equal to the average rate of change.

5(16pts) Find the absolute extrema of the function $f(x) = x^3 + x^2 - 8x + 1$ in the interval $[-3, 1]$. (Read-out answers from calculators receive no point. Show all the works.)

6(18pts) The first and second derivatives of the function

$$f(x) = \frac{(x+1)^2}{1+x^2}$$

are given as follows:

$$f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}, \qquad f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3}.$$

Sketch a graph of the function $y = f(x)$, including all significant features of the functions: domain of the function, x-intercept, y-intercept, vertical and horizontal asymptotes, local extrema, interval of increasing and decreasing, inflection points, interval of concavities. A calculator assisted graph without accompanying analysis receive no credit.

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2 Bonus Points: Newton's Method is used to (a) find the solution of an equation, (b) find the derivative of a function, (c) find the inverse of a function, (d) all the above. (... *The End*)