Math 106-350 Spring '13 **Exam 3** **Score:** _____

Name: _____ **TA's Name:** _____

Instructions: You must show supporting work as much as possible to receive full and partial credits. No text book, notes, formula sheets allowed.

1. (**10 pts**) Consider the integral

$$\int_0^1 e^{x^2} dx$$

(a) (5 pts) Estimate the value using the left-hand sum with $n = 2$, i.e. L_2. Show your work.

(b) (10 pts) Use calculator to estimate the value using the left-hand sum with $n = 20$, i.e. L_{20}. Is your estimate an overestimate or underestimate and why?

2. (**10 pts**) If $f(2) = 1$, $\int_1^2 f(x)dx = 5$, $\int_1^3 f(x)dx = 3$, $F(x) = \int_0^x f(t)dt$, find the following values:

(a) $\int_2^3 f(t)dt$ (b) $\int_1^2 (3f(t) + 2t)dt$ (c) $F'(2)$

3. (**10 pts**) (a) Verify that $x(t) = 3 - 2e^{-t}$ is a solution to the differential equation $x'(t) = 3 - x(t)$ satisfying $x(0) = 1$.

(b) Find all solutions to the differential equation: $\dfrac{dy}{dx} = \dfrac{1}{1 + x^2}$.

(*Page 1. Continue on Next Page ...*)

4. (**15 pts**) Find the exact value or integral of

(a) $\int 3x^2 e^{x^3+1} dx$

(b) $\int_0^9 \frac{1}{\sqrt{\sqrt{x}+1}} dx$

(c) $\int \frac{2x+1}{1+x^2} dx$

5. (**10 pts**) (a) A particle is moving from $(2, 2)$ to $(2, 0)$ in 1 second in a straight path. Write a parameterization for its motion.

(b) A particle is moving around a circle of radius 3 in from $(3, 0)$ back to itself in 2 seconds. Write a parameterization for its motion.

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6. (**15 pts**) Find the limit, using L'Hopital's Rule if applicable.

(a) $\lim_{x\to 0} \dfrac{\cos x - 1}{x^2}$

(b) $\lim_{z\to \infty} z^2 e^{-z}$

(c) $\lim_{x\to \infty} x^{1/x}$

(d) $\lim_{x\to 0} \left[\dfrac{1}{x} - \dfrac{1}{\sin x}\right]$

7. (**10 pts**) If F is an antiderivative of f, i.e. $F'(x) = f(x)$, and $F(1) = 2,\ F(3) = 0$, find the following values:

(a) The definite integral $\displaystyle\int_1^3 f(x)dx$

(b) $\displaystyle\int_1^3 [2f(x) + 1]dx$

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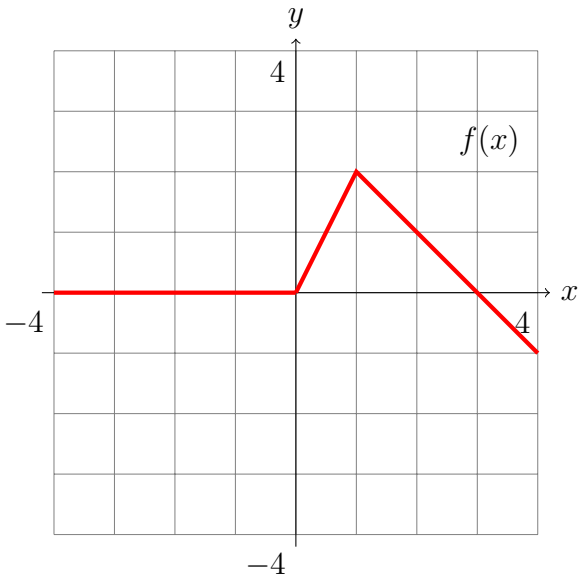
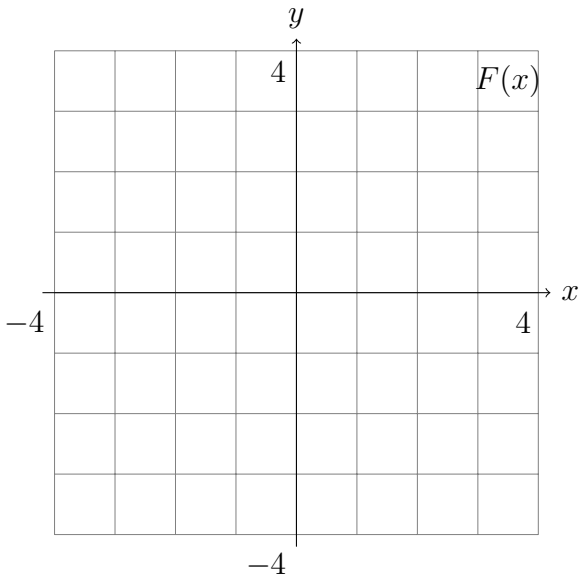
8. (**10 pts**) The graph of a function f is given below.





(a) In the blank coordinate frame sketch the antiderivative F of f if $F(-2) = -1$.

(b) What are the exact values of $F(0),\ F(1),\ F(3),\ F(4)$?

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9. (**10 pts**) Find the following derivatives.

(a) $\dfrac{d}{dx}\displaystyle\int_1^x \sin(t^2)dt$

(b) $\dfrac{d}{dx}\displaystyle\int_1^{x^2+1} \sin(t^2)dt$

(c) Find all critical points of the function $F(x) = \displaystyle\int_{x^2}^{\pi} \frac{\sin t}{t}dt$

(d) Find all critical points of the function $G(x) = \displaystyle\int_{x^2}^{4x^2} e^{\sqrt{z}}dz$. What is the value of $G(0)$? Is $G(0)$ a local minimum, a local maximum, or neither?

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2 point Bonus Question: The state fish of Nebraska is: (a) channel catfish; (b) rainbow trout; (c) snakehead; (d) bluegill.