Math106 Fall 2013
Calculus I University of Nebraska-Lincoln

Name: **Section/Instructor:**

(print legibly)

Final Exam

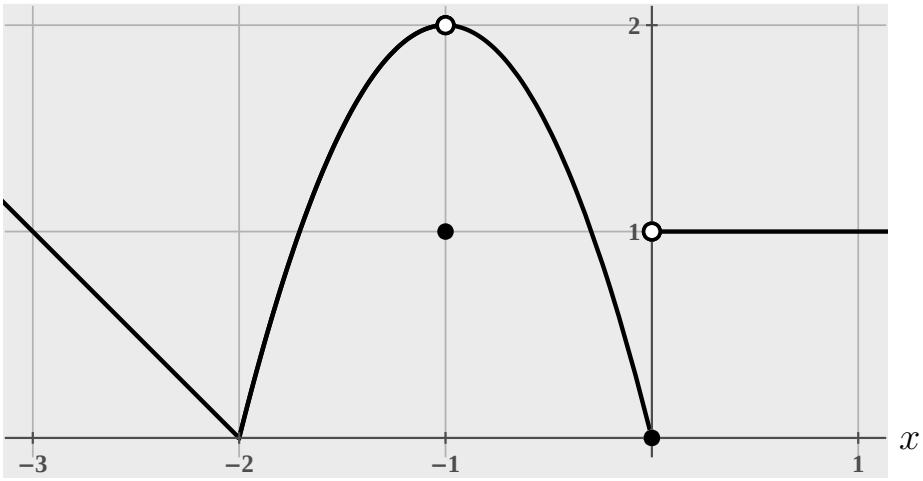
Instructions

- Turn off all communication devices. If you do not do so, then you will not receive any credit for your exam.
- There are 11 pages in this exam with 10 problems. Before you begin, make sure that your exam has all 11 pages.
- The examination period is from 6pm to 8pm. If you wish to receive credit for your exam, then make sure that your exam is submitted for grading by 8pm.
- During the exam, you may use a calculator, with no built-in CAS. No notes, text or reference may be used.
- To receive full credit for a problem, you must provide a correct answer and a sufficient amount of work so that it can be determined how you arrived at your answer.
- Clearly indicate what your solutions are and any work that you do not want to be included in the grading process.
- Write your solutions in an explicit form whenever possible. Unless indicated otherwise, your answers must be exact, not a numerical approximation.
- If you wish to speak with a proctor during the exam, then raise your hand and a proctor will come to you.
- Each problem will be graded out of 20 points.
- If it is determined that you have given or received any unauthorized aid during this exam, then you will receive no credit for your exam.

Problem	Score
1	
2	
3	
4	
5	

Problem	Score
6	
7	
8	
9	
10	
Total	

1. (20 points) The following problems refer to the graph of $y = f(x)$ in the figure. If a limit does not exist, explain why.



(a) Evaluate $\lim_{x \to 0^+} f(x)$. If it does not exist, explain why.

(b) Evaluate $\lim_{x \to 0^-} f(x)$. If it does not exist, explain why.

(c) Evaluate $\lim_{x \to 0} f(x)$. If it does not exist, explain why.

(d) Evaluate $\lim_{x \to -1} f(x)$. If it does not exist, explain why.

(e) At which x-values in the interval $(-3, 1)$, if any, is the function f **not** continuous? (List all of them.)

(f) At which x-values in the interval $(-3, 1)$, if any, is the function f **not** differentiable? (List all of them.)

(g) Provide the value of $f'(-2.5)$.

2. (20 points) Provide a careful sketch of a function $y = f(x)$ that satisfies all the following conditions. No formula is needed– just carefully sketch and label your graph. Full credit will not be given for graphs that do not clearly satisfy the conditions indicated.

Conditions:

(a) $f(x)$ is continuous at all $x \neq 0$;

(b) $\lim_{x \to 0^-} f(x) = -\infty$ and $\lim_{x \to 0^+} f(x) = -\infty$;

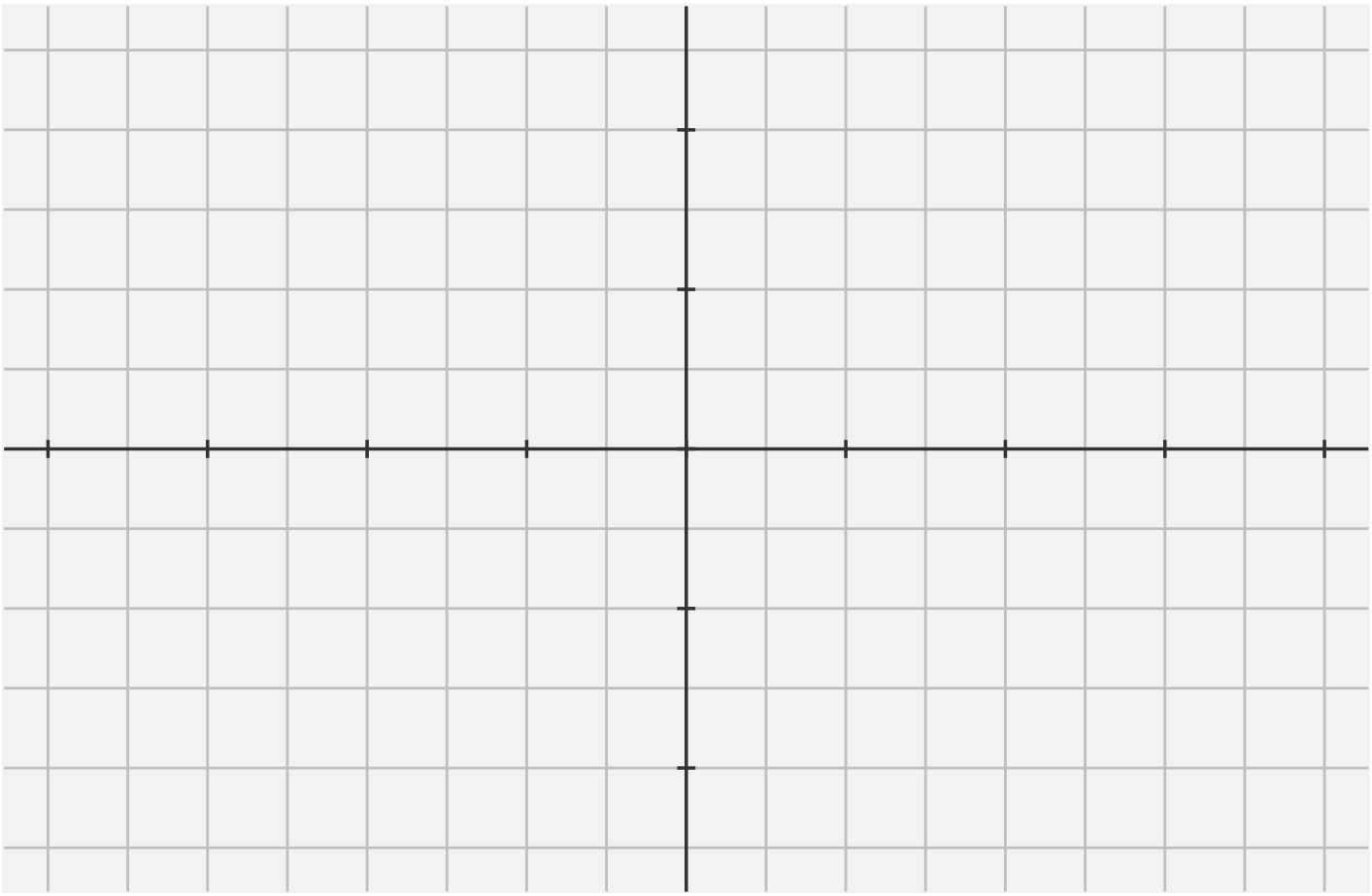
(c) $f'(0)$ is undefined and $f'(x)$ exists for all $x \neq 0$;

(d) $f'(1) = 0$

(e) $f'(x) < 0$ for all $x < 0$ and for all $x > 1$; and $f'(x) > 0$ for $0 < x < 1$.

(f) $f''(x) < 0$ for $x < 0$ and for $0 < x < 2$; and $f''(x) > 0$ for all $x > 2$.

In your sketch, indicate all local extrema, critical points, inflection points and asymptotes.



3. (20 points) **Use the definition of a derivative** to find $f'(x)$ for the function

$$f(x) = 3x^2 - 1.$$

Include all your work. No credit will be given if insufficient work is provided or the definition of the derivative is not used.

4. (20 points) A ruptured oil tanker causes a circular oil slick on the surface of the ocean. When the radius is 200 meters, it is expanding at a rate of 0.2 meters/minute. What is the rate at which the area of the oil slick changing? Include units with your answer.

5. (20 points) For this problem $f(x) = 4x - 10x^{2/5}$

   (a) Find all the critical numbers for $f(x)$.

   (b) Find the global maximum and global minimum for $f(x)$ over the closed interval $[-1, 32]$.

6. (20 points) A company needs to produce a square bottomed rectangular box with a volume of 4 cubic feet. The box has no top, just sides and a bottom. Provide the dimensions of the box that uses the least amount of material (has minimal total surface area). Provide an exact result. Include units with your answer. Justify your answer.

7. (20 points) The following problems refer to this parametric curve:

$$x = t^4 - 2t \quad \text{and} \quad y = 4t + 2t^3.$$

(a) Find $\dfrac{dy}{dx}$ as a function of t.

(b) Find the equation of the tangent line to the parametric curve above at the point where $t = 1$.

8. (20 points) Evaluate the following limits. If a limit does not exist, explain why.

(a) $\lim_{x\to\infty} \dfrac{4e^{6x}}{2e^{6x} - 5e^{x}}$.

(b) $\lim_{x\to 3^-} \dfrac{x^2}{3 - x}$

(c) $\lim_{x\to 0} \dfrac{\sin(x) - x}{6x^2}$.

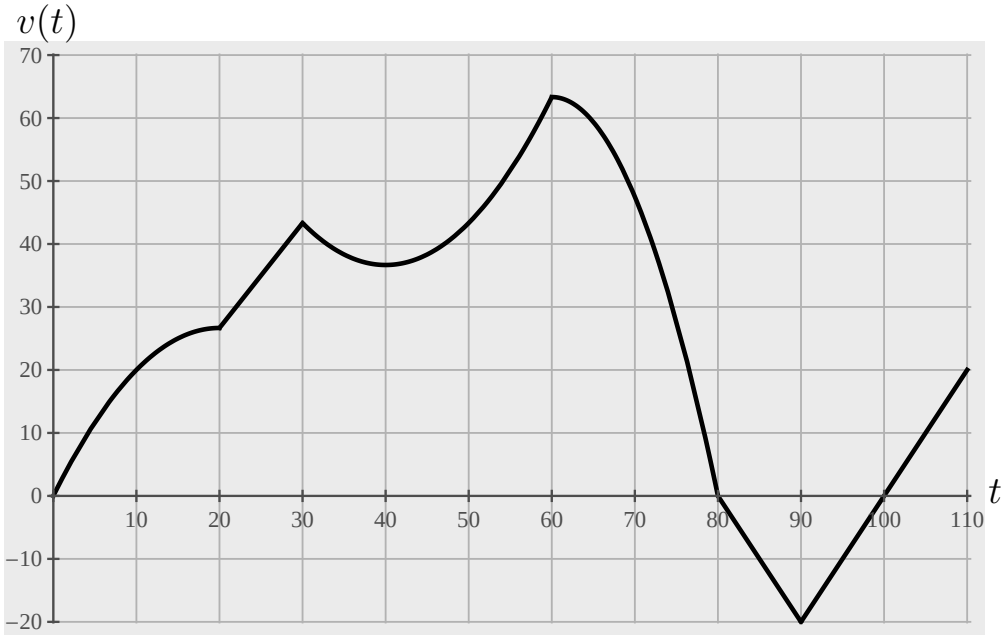
9. (20 points)

(a) Evaluate $\int_1^2 \left(\frac{1}{x^2} + x - \frac{1}{2}\right)\, dx.$

(b) Evaluate $\int x^2 \left(1 + x^3\right)^5\, dx.$

(c) Evaluate $\frac{d}{dx} \int_1^{x^2} \sqrt{1+t}\, dt.$

10. (20 points) A rocket's **vertical velocity** $v(t)$, in meters per second at time t seconds, is described in the graph below. The following problems refer to this graph. Explain your answers and include units.



(a) During what time interval(s) is the rocket heading upward?

(b) At what time is the rocket at its maximum height?

(c) During what time interval(s) does the rocket have a positive acceleration?

(d) Produce an integral that provides the rocket's total change in position from time $t = 0$ seconds to time $t = 110$ seconds.