

MATH 104    HOUR 3a    PRINT NAME \_\_\_\_\_



March 31, 2006    SIGNATURE \_\_\_\_\_

---

YOU MAY NOT SHARE CALCULATORS. SHOW ALL YOUR WORK.

Cross out work that you don't want graded. Circle your answers.

---

(8) 1. Find an equation of the tangent line to the graph of  
 $y = f(x) = \ln(x^2 e^{2x})$  at the point  $(1, 2)$ .

Page	Points
1-25	
2-25	
3-25	
4-25	
TOTAL	

(8) 2. If  $g(x) = x^2 \ln(2x)$ , find  $g''(e)$ .

(9) 3. Given the cost function  $C = C(x) = 4x^2 + 21x + 576$  dollars, use calculus methods to determine the number of units  $x$  that should be produced in order to minimize the average cost per unit.

(10) 4. Let  $y = f(x)$  be a function such that  $f''(x) = x^3(x + 6)(x - 5)^2$  for all  $x \in (-\infty, +\infty)$ .

(a) List the open interval(s) where the graph of  $f$  is concave up. (Hint: Chart  $f''$ )

(b) List the number(s)  $x$  where  $(x, f(x))$  is a point of inflection on the graph of  $f$ .

(15) 5. Let  $y = f(x) = (x + 1)^3(x + 3)$ .

(a) Find all critical points of  $f$ .

(b) Determine the intervals of concavity and the inflection points for  $f$ .

(c) Determine the relative maxima and relative minima of  $f$ .

(10) 7. If  $f(x) = xe^{-\frac{x^2}{2}}$ , the derivative is  $f'(x) = e^{-\frac{x^2}{2}}(1 - x^2)$ . Use this information to find all critical points (or critical numbers) of the graph  $y = f(x)$ . For each critical point, say whether it is a relative minimum, a relative maximum, or neither. (Hint: Chart  $f'(x)$ ).

(15) 8. Suppose that a manufacturer can sell  $x$  widgets at a price of  $80 - .02x$  dollars each and assume that it costs  $40x + 1500$  dollars to produce all  $x$  of them.

(a) Find the revenue function, and the profit function.

(b) Determine the value of  $x$  which will maximize the revenue function.

(c) Determine the value of  $x$  that will maximize the profit function.

(6) 9. If the total profit function is modeled by  $P = 0.003x^2 + 0.019x - 1200$ , use differentials to approximate the change in profit corresponding to an increase in sales of one unit when  $x = 600$ .

(8) 10. Find two nonnegative numbers  $x, y$ , whose difference is 75 and whose product is a minimum.

(6) 11. Use calculus methods to find the absolute maximum value  $M$  and the absolute minimum value  $m$  of the function  $f(x) = x^3 - 12x + 4$  on the closed interval  $[0, 3]$ .

(5) 12. Use differentials to approximate  $\sqrt{47}$