MATH 104 HOUR 4 REVIEW

1. Find the following antiderivatives: Whenever you use the substitution method, clearly indicate what substitution u is being used. Show all your work.

(a) $\int \left(2x + \frac{1}{x}\right)^2 dx$ (b) $\int \left(x^{-2/5} + \frac{3}{e^{4x}} + 100\right) dx$

(c) $\int \left(xe^{-x^2} + \frac{3x}{x^2+5}\right) dx$ (d) $\int \frac{e^{3/x}}{x^2} dx$

(e) $\int \left(x^{-1/2} + \frac{2}{e^{2x}} - 5x^{-1}\right) dx$ $(f) \int \left(\frac{3x^3+2}{2x^4}\right) dx$

2.Evaluate the following definite integrals:

(a) $\int_{-1}^{1} \left(6x + 3x^2\right) dx$; (b) $\int_{0}^{1} \left(4e^{2x} - 2x\right) dx$

(c) $\int_{0}^{2} \left[x^2 - e^{3x}\right] dx$; (d) $\int_{1}^{3} \left(1 + \frac{1}{x}\right) dx$

3. (a) Find an equation of the tangent line to the graph of the curve $y = f(x) = \left(x^2 - 2\right)^3$ at the point $(2, 8)$.

(b) Find an equation of the tangent line to the graph of the curve $y = f(x) = \frac{e^{3x}}{x^2+1}$ at the point $(0, 1)$.

4. Find the cost function $C(x)$ if the marginal cost function is
$C'(x) = 6x^2 - 6x + 30$ dollars and $C(2) = \$132$.

5. Approximate the area under the graph of the following functions on the interval $[0, 2]$, with $n = 4$, using (a) left end points; and (b) right endpoints:

(i) $f(x) = \sqrt{x^2+1}$

(ii) $f(x) = e^{x^2}$

6. If $\int_1^{10} g(x)dx = 12$ and $\int_1^5 g(x)dx = 4$, evaluate the following definite integrals:

(a) $\int_5^1 2g(x)dx$

(b) $\int_5^{10} g(x)dx$

(c) $\int_1^{10} (2x - g(x))dx$

7. Find the indefinite integrals:

(a) $\int \frac{\ln x}{x} dx$. (b) $\int (3x^2 + 9)(x^3 + 9x + 201)^{17} dx$

8. Let $p = D(x) = 30 - 2x$ dollars be the **demand** function and let $p = S(x) = x^2 + 6$ dollars be the **supply** function for some commodity.
(a) Find the **equilibrium point** (x_0, p_0).

(b) Sketch a graph of the supply and demand curves and shade the region whose area is equal to the **Producer Surplus**, and the **Consumer Surplus**.

(c) Find the **Producer Surplus**.

9. Let R be the region in the **first quadrant** enclosed by the curves $y = e^x$, $y = e$, and $x = 0$.

(a) Sketch a graph of the region R including the **intersection points** of the above curves.

(b) Find $A(R)$, the area of R.

10. Let R be the region bounded by the graphs of $y = 2x$ and $y = x^2 - 3$.

(a) Sketch a graph of the region R including the **intersection points** of the curves.

(b) Find $A(R)$, the area of R.

11. Let R be the region bounded by the graphs of $y = |x|$ and $x^2 + y^2 = 2$.

(a) Sketch a graph of the region R including the **intersection points** of the curves.

(b) **Express** $A(R)$, the area of R, in terms of definite integrals. (Do **not** evaluate the integrals.)

12. Find the area between the graphs of $y = 1 - x^2$ and $y = x - 1$.

13. A deposit of \$2000.00 is made in a savings account at an annual interest rate of 6% compounded continuously. Find the **average balance** in the account during the first 5 years.

14. Find $f(x)$ if $f''(x) = 36x + 4$, $f(0) = 2$ and $f'(0) = -4$.