Letter to the Editor


A R T I C L E  I N F O

Keywords:
Population dynamics
Discrete time mathematical models

This publication is devoted to the problem of application of one of the physical principles to ecological modeling – to the processes of selection and/or constructing mathematical models of population dynamics. In modern literature it is easy to find a lot of good and bad examples of application of various physical ideas, methods, apparatus etc. to solution of various ecological problems. From that point of view it looks like a normal way for the development of ecological modeling. And it looks rather new – I’ve never heard about application of Time Invariant Principle to ecological problems. On the other hand, the results, which were announced in this manuscript, are extraordinary and revolutionary. During several centuries (formally since 1242, Fibonacci) investigators use discrete time mathematical models for the description of population dynamics, and they couldn’t imagine themselves that it isn’t correct from the standpoint of one of the basic physical principles.

It is well known that every revolution is a result of big mistake(s). It leads to the necessity to check carefully the first chapters of manuscript with basic ideas and assumptions, which could lead to revolutionary results.

On the page 2 of manuscript we see: “Time Invariance Principle (TIP): A physical law has the same mathematical form to every independent choice of observation time.” And on the page 3 we have: “Therefore,

\[ \phi_n(x_0, \tau) = \phi^n(x_0, \tau) \]

that is, the \( n \)th iterative composition of \( \phi_1 \) must have the same functional form as itself since both \( \phi_1 \) and \( \phi_n \) have the same functional form. This property can be used as a diagnostic test for probable TIP-nonconformity.”

Between these parts of the manuscript the author presented two lemmas and one theorem about the impossibility to deduce discrete logistic model from one-dimensional (autonomous or non-autonomous) ordinary differential equation. First of all, ordinary differential equations (ODEs) are not the best mathematical apparatus, which is normally used for the description of population dynamics with seasonal development. It is well known that many discrete time dynamical systems do not fit within the framework of ODEs. However, it is also well known that most discrete time population dynamics models (in particular, discrete logistic model) can be obtained from differential equations with impulses (Nedorezov and Nedorezova, 1995; Geritz and Kisdi, 2004).

Second, it is necessary to say some words about “TIP-conforming property” (page 2, (1)). For this reason let us consider the model

\[ x_{k+1} = F(x_k), \]  

where \( F \) is non-negative continuous map of \( R^n \) into itself. It is well known that iterations of this map \( F \) form the semigroup (there is the absence of inverse elements for Moran–Ricker map, for discrete logistic map etc.) with Abel’ property:

\[ F^n(F^m(\cdot)) = F^{m+n}(\cdot), \]  

The author forgot to point out this (TIP-conforming!) property, which is realized for all well known discrete time models. There is one difference between the original formula (relation (1) in the manuscript) and relation (2): in the original definition it is assumed that \( n \) and \( m \) are real but in (2) \( n, m \in N \) (integer values). But for discrete models of (1) type we cannot a priori have a family of maps with continuous parameters. It means that if we postulate author’s TIP-conforming property, then it immediately leads to the situation when all other models become TIP-nonconforming. Consequently, all “results” about non-applicability of discrete models were postulated.

Third, we have to read carefully Time Invariance Principle and understand what we can only conclude (from the formulation of this principle) and what we cannot conclude. Let us present model (1) in the following form:

\[ x_{k+m+1} = F^m(x_k), \]

where \( k \) plays the role of absolute time, \( m \) is the length of time interval between two time moments. From TIP we can conclude that \( F^m(\cdot) \) does not depend on \( k \) only. The value of \( F^m(\cdot) \) depends on initial values of population size and model parameters but it does not depend on absolute time. On the other words, for every \( k \) we have the same law of population size changing in time. And, moreover, if we read the Principle carefully we can find that there are no words about the equivalence of functional forms. It allows us to conclude that one of the most important author’s problems is in the presentation of discrete models: if we present in the form when we have initial values, model parameters, absolute time and relative
time we will have immediately that there are no contradictions between discrete time models and Time Invariance Principle.

Fourth (Page 3, Table 1) it was pointed out in Table 1 that the Leslie matrix model (Leslie, 1945, 1948) belongs to group of probable TIP-nonconformity models. Let us assume that it is true. If so, the author has to explain what are the names of physical laws or fundamental physical principles, which were violated at any step of model construction? Moreover, if Leslie matrix model is very bad—the author must show how we can construct a better model of population dynamics with age structure?

It is well known that Leslie matrix model is based on the following balance assumptions. Let \( \vec{N}_t = (N^0_t, \ldots, N^k_t) \) be a vector of population state at time \( t \), \( N^j_t \) is the number of individuals of age \( j \) at the same time moment. After one year survived individuals become older by one year, so we have the following relation:

\[
N^{j+1}_t = s_j N^j_t.
\]

At that moment it is very important to ask the author—what is the name of physical law or fundamental physical principle, which was violated by this balance relation? Can the author any present other relation for the description of changing of individuals of age group during one year?

During this time period individuals produce new individuals of the youngest age group:

\[
N^0_{t+1} = \sum_{i=1}^{k} f_i N^i_t.
\]

The situation looks rather strange—we did not violate any physical law and/or principle, but the model cannot be used for the description of population dynamics? It allows me to conclude that the basic problem of manuscript is in the application of physical principle to ecological models.

Fifth (Page 3, Table 1), the generalized Beverton—Holt model is presented in the following form:

\[
N_{t+1} = \frac{b N_t}{1 + (h N_t)^\gamma}.
\]

(Maynard Smith and Slatkin, 1973; Hassell, 1975; Hassell et al., 1976). I did like to note that in modern literature this model is called Maynard Smith—Slatkin model; Hassell model has the following form:

\[
N_{t+1} = \frac{b N_t}{(1 + h N_t)^\gamma}.
\]

It is easy to understand that these models are different. Moreover, the real author of Beverton and Holt (1957) model is Skellam (1951). But before Skellam’ publication this model was described in monograph by V.A. Kostitzin (1937).

Finally, in this manuscript we can find several logical mistakes and some other mistakes, which depend on insufficient information about current conditions in ecological modeling. I do not want to talk about the two main parts of manuscript—Abstract and Introduction, which contain a lot of incorrect phrases and actual mistakes too. I only hope that author and reviewers (who had recommended the manuscript for publication) will find good answers onto the questions pointed out above.

References


Lev V. Nedorezov
Forest-Technical Academy, Saint-Petersburg, Russia
E-mail address: l.v.nedorezov@gmail.com

23 December 2008
Available online xxx