Math 208 Final Exam **Name** ______________________________.

1. (14 points) Find and classify the critical points of

$$f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8.$$

2. (14 points) Find an equation of the tangent plane to the surface $\cos(\pi x) - x^2 y = 4 - yz - e^{xz}$ at the point (0,1,2).

3. (14 points) Evaluate the triple iterated integral

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2)\, dzdxdy.$$

4. (14 points) Evaluate the double iterated integral

$$\int_{-1}^0 \int_{-\sqrt{1-y^2}}^0 \frac{2}{1 + \sqrt{x^2 + y^2}} dxdy.$$

5. (14 points) Evaluate the iterated integral

$$\int_0^3 \int_{y^2}^9 y\cos(x^2)\,dx\,dy.$$

6. (16 points)

(a) If $w = w(x)$, and $x = x(r, s, t)$. Write down a chain rule formula for $\frac{\partial w}{\partial s}$.

(b) Find $\vec{curl} < xy, yz, xz >$.

7. (14 points) Use Green's theorem to find the work done by the force field $\vec{F}(x, y) = (y^2, x^2)$ in moving an object around the triangle with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$ one time in the counterclockwise direction.

8. (16 points) Given the force field $\vec{F}(x, y, z) =< y^2 \cos(xy) - 3, \sin(xy) + 2y + xy \cos(xy), 2z >$.

a. Find a potential function for this force field.

b. How much work is done by this force field in moving an object from (2,-1,1) to (1,3,0)?

9. (14 points) Find the flux of the vector field $\vec{F}(x, y, z) =< -xz, -yz, z^2 >$ up through the cone $z = \sqrt{x^2 + y^2}$ for $0 \leq z \leq 4$, oriented upward.

10. (14 points) A rectangular box without a lid is made from 12 square feet of cardboard. Use Lagrange multipliers to find the maximum volume of such a box.

11. (14 points) Given $f(x,y) = \sqrt{x^2 + y^2}$.

a. Find the rate of change of f at (3,4) in the direction from the point (3,4) to the point (0,0).

b. Find the maximum value of all of the directional derivitives of f at (3,4).

c. Find a vector $\vec{w}$ such that the rate of change of f at (3,4) in the direction of $\vec{w}$ is zero.

12. (14 points) Find the quadratic approximation of $f(x,y) = 2 + 3x + 4y + x^2 - 2y^2$ near $x = 2$, $y = 1$.

13. (14 points) Find the equation of the plane passing through the points $(2, 1, -3)$, $(1, 2, 1)$, and $(1, -2, 4)$.

14. (14 points) Use Stoke's theorem to evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y, z) =< 0, x^2, 0 >$ and C is the intersection of the plane $2x + y + z = 2$ with the coordinate planes traversed counter-clockwise as viewed from above.