1. (14 points) Given $f(x, y) = x^2 + 3x^2y^3 - y^3$.

   (a) Find the directional derivative of $f$ at $(1, 2)$ in the direction from $(1, 2)$ towards the point $(2, 1)$.

   (b) Find the minimum value of the directional derivatives of $f$ at $(1, 2)$.

   (c) Find a unit vector $\vec{v}$ so that the rate of change of $f$ at $(1, 2)$ in the direction of $\vec{v}$ is zero.

2. (12 points) Find an equation of the tangent plane and normal line to the surface $x^2 + 2y^2 = 27 - z^2$ at the point $(4, -1, 3)$. 

3. (12 points) Find and classify the critical points of \( f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy \).

4. (12 points) Find an equation of the plane passing through the point \( P = (1, 0, 2) \) and perpendicular to the line of intersection of the planes \( z = 2x - 3y + 4 \), \( z = x + y + 6 \).
5. (15 points) Evaluate each of the following integrals:

(a) \( \int x^3 \ln x \, dx \)

(b) \( \int x \sin(3x) \, dx \)

(c) \( \int \frac{3x^2 - 2x + 5}{(x+1)(x^2+4)} \, dx \)

6. (12 points) Evaluate the following:

(a) \( \lim_{n \to \infty} (1 - \frac{2}{n})^3n \)

(b) \( \int_1^\infty \frac{x}{(8+x^2)^2} \, dx \)
7. (12 points)

(a) Find a parametric representation of the part of the ellipse \( \frac{x^2}{25} + \frac{y^2}{4} = 1 \) in the upper half-plane that goes from \((0, 2)\) to \((-5, 0)\).

(b) Given \( w = f(x, y, z) \) and \( x = g(r, s, t), y = h(r, s, t), z = k(r, s, t) \). Write down a chain rule formula for \( \frac{\partial w}{\partial r} \).

8. (12 points) Determine if each of the following infinite series converge or diverge. Give reasons for your answers.

(a) \( \sum_{k=1}^{\infty} \frac{k+1}{3k+100} \)

(b) \( \sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}} \)
9. (12 points) Find the interval of convergence for the power series \( \sum_{k=1}^{\infty} \frac{(-1)^k 2^k x^k}{k} \).

10. (11 points) Use Lagrange multipliers to find the point on the plane \( 2x + y - z = 5 \) closest to the origin.
11. (12 points) Evaluate the following double iterated integral $\int_0^1 \int_y^1 x^2 e^{xy} \, dx \, dy$ by reversing the order of integration.

12. (12 points) Evaluate the double iterated integral $\int_{-1}^0 \int_0^{\sqrt{1-x^2}} 5\sqrt{x^2+y^2} \, dy \, dx$. 
13. (12 points) Using a triple integral, find the volume of the solid which is bounded above by the sphere $x^2 + y^2 + z^2 = 16$ and bounded below by the upper nappe of the cone $z^2 = 3(x^2 + y^2)$.

14. (14 points)

(a) Find the work done by the force field $\vec{F}(x, y) = <2x + y, 1 + x>$ to move an object along the line segment from $(1, 2)$ to $(2, -3)$.

(b) Find the Taylor polynomial of degree three about $x = 0$ for $f(x) = \sqrt{1 - 2x}$. 
15. (14 points) Given that \( \vec{r} = \langle \cos t + t \sin t, \sin t - t \cos t \rangle \) is the position vector of an object at time \( t \). Find the velocity, speed, acceleration, normal component of acceleration, and tangential component of acceleration at time \( t \). Find the radius of curvature of the curve at time \( t \).

16. (12 points) Given \( \vec{u} = \langle 2, 1, 3 \rangle \), \( \vec{v} = \langle -1, 2, 1 \rangle \)

(a) Find the angle between \( \vec{u} \) and \( \vec{v} \).

(b) Find the vector projection of \( \vec{u} \) on \( \vec{v} \).