Math 208 (Exam 3) **Name** ______________________.

1. (15 points) Given $f(x,y,z) = 2x^2y^3z^4$ and $\vec{F}(x,y,z) =< x + 2y + 3z, yz, xyz >$, find the gradient of $f(x,y,z)$, the divergence of $\vec{F}(x,y,z)$, and the curl of $\vec{F}(x,y,z)$.

2. (12 points) Evaluate the line integral $\int_C 2xydx + (x+y)dy$ where C is the line segment from $(2,-1)$ to $(-2,1)$.

3. (20 points)

(a) Without finding a potential function, show that the vector field $\vec{F}(x,y) =< xy^2+x^2, x^2y+2y >$ is conservative.

(b) Without finding a potential function, show that the vector field $\vec{F}(x,y,z) =< y^2z + \cos x, 2xyz + 3z^2, xy^2 + 6yz >$ is a gradient vector field.

(c) Find a potential function for the vector field $\vec{F}(x,y,z) =< y^2z + \cos x, 2xyz + 3z^2, xy^2 + 6yz >$.

(d) Find the work done by the force field $\vec{F}(x,y,z) =< y^2z + \cos x, 2xyz + 3z^2, xy^2 + 6yz >$ in moving an object from the point $(0,2,1)$ to the point $(1,2,-1)$.

4. (13 points) Evaluate the surface integral $\int\int_S \frac{z}{\sqrt{1+4x^2+4y^2}} dS$, where S is the part of the surface $z = 2 + x^2 + y^2$ inside the cylinder $x^2 + y^2 = 1$.

5. (14 points) Use Green's theorem to find the work done by the force field $\vec{F}(x, y) =< -y^3, x^3 >$ in moving an object around the circle $x^2 + y^2 = 16$ one time in the clockwise direction.

6. (13 points) Find the flux of the vector field $\vec{F}(x, y, z) =< x^3, y^3, z^3 >$ out of the closed surface bounding the solid region $x^2 + y^2 \leq 4$, $0 \leq z \leq 2$, oriented outward.

7. (13 points) Use Stoke's theorem to find the circulation of the vector field $\vec{F}(x, y, z) =< y, z, x >$ about the curve C which is the triangle with vertices $(2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, -2)$ traversed in that order (this triangle lies in the plane $3x + 2y - 3z = 6$).