1. (30 points) Evaluate each of the following:
   (a) $\int_0^1 \int_{x^2}^x (2xy + x^2 + 4y^3) \, dy \, dx$
   (b) $\int_0^2 \int_{\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2 + y^2)^3 \, dy \, dx$
   (c) $\int_0^1 \int_x^1 e^{y^2} \, dy \, dx$
2. (16 points) Find the directional derivative of \( f(x, y, z) = x^2yz + y^2 - z \) at the point \( P = (2, 1, 3) \) in the direction from \( P \) to \( Q = (3, 0, 1) \). Find the value of the maximum directional derivative at \( P \). Find a unit vector \( \vec{v} \) so that the directional derivative of \( f \) at \( P \) in the direction of \( \vec{v} \) is zero.

3. (14 points) Find and classify all critical points for \( f(x, y) = x^3 - y^3 - 2xy + 6 \).
4. (14 points) Find the points on the sphere \( x^2 + y^2 + z^2 = 30 \), where \( f(x, y, z) = x - 2y + 5z \) has its maximum and minimum values.

5. (13 points) A triangular plate has vertices at (0, 0), (1, 2), and (1, -1). If the mass density of the plate at \((x, y)\) is given by \( f(x, y) = x^2 + 4y^2 \) with units mass per unit area, find a double iterated integral that gives the mass of the plate. Do not evaluate this integral.
6. (13 points) Write the triple integral $\int \int \int_Q f(x, y, z) \, dV$, where $Q$ is the region bounded by the coordinate planes and the plane $2x + y + 3z = 6$, as a triple iterated integral.