Due: April 2nd


3. Do Exercise 9.1.M in the text, which should read:
   
   (a) Give an example of a decreasing sequence of closed balls in a complete metric space with empty intersection. Compare with Exercise 7.2.J.
   
   HINT: Use a metric on \( \mathbb{N} \) topologically equivalent to the discrete metric so that \( \{n \geq k\} \) are closed balls.
   
   (b) Show that a metric space \((M, d)\) is complete if and only if every decreasing sequence of closed balls with radii going to zero has a nonempty intersection.


5. (June 1998 Qual) Do only part (b) of this question; part (a) is included only for your reference.
   
   (a) Define the term compact set in a metric space.
   
   (b) Let \((X, \rho)\) be a metric space and \((x_n)\) a sequence in \(X\) that converges to \(a \in X\). Prove directly from the (open cover) definition of compactness that \(K := \{a\} \cup \{x_n : n \geq 1\}\) is compact.